# Nonlinear Characterization of Electrostatic MEMS Resonators


Manu Agarwal, Kwan Kyu Park, Rob N. Candler[1], Bongsang Kim, Matthew A. Hopcroft, Saurabh A. Chandorkar, Chandra M. Jha, Renata Melamud, Thomas W. Kenny and Boris Murmann

Departments of Electrical and Mechanical Engineering
Stanford University
Stanford, California, USA
e-mail: amanu@stanford.edu



***Abstract*— Encapsulated micromechanical resonator technology is becoming important as a potential replacement for quartz for several applications. In this work we report the nonlinear characterization, particularly the A-f effect, in these resonators. The A-f effect in quartz has been well studied in the 1970's and 1980's [1, 2], as it dictates the maximum power (current) that can be handled by the resonator. MEMS resonators tend to have a strong A-f effect compared to quartz, and this is the reason for the low power handling in these devices in comparison to quartz crystal resonators. In this work we report the mechanism of nonlinearities in these devices and find design guidelines to improve performance.**


## I. Introduction

Electrostatic silison MEMS resonators and filters have become a promising solution for frequency references and signal processing applications in recent times [3]. Integration of these micromechanical structures with on-chip CMOS circuitry potentially leads to considerable size and cost reduction, making them attractive and viable for commercial applications.

The resonators in this work are fabricated in single crystal silicon wafers and encapsulated with epitaxially deposited polysilicon. The structures are defined by deep reactive-ion etching in a silicon-on-oxide wafer, released with a HF vapor etch, and then encapsulated with epitaxial deposition [4]. The resonators are sufficiently robust for standard IC dicing and handling, and show low aging [5].

Several efforts to commercialize electrostatic MEMS resonator technology are currently underway. Some of the interesting applications of commercial interest are telecommunications, timekeeping and networking. MEMS resonator based oscillators, with phase noise performance that satisfies the GSM standard requirements, have already been reported [6, 7]. Resonators with frequencies in the GHz regime, with A-f products $>10^{13}$ have also been developed[8, 9]. Low vibration sensitivities[10] and active and passive control of temperature sensitivity are being investigated.

For most of these applications, nonlinearities in the resonators limit the ultimate short term frequency stability that can be achieved. In sensors, this stability is a measure of the achievable resolution. Nonlinearities in quartz have been studied and their impact on frequency stability of oscillators has been previously discussed [1]. Due to the amplitude-frequency dependence (A-f) effect, the nonlinearities cause Duffing bifurcation instability and/or amplitude fluctuation induced frequency fluctuation type effects [11, 12], thereby limiting the useful signal strength available from these devices. The generating mechanisms for nonlinearities that can cause this effect in MEMS have been developed previously [7, 11]. Nonlinear multiplication of low frequency electronic noise into the close-to-carrier noise has also been studied [13-15].

In this work we present the modeling and analysis of the A-f effect in MEMS resonators with experimental verification using a double-ended tuning fork resonator, shown in Figure 1.

## II. A-f Effect in MEMS Resonators

Several works have explored different limits on the maximum amplitude of oscillation for electrostatic resonators [16]. As in quartz, the A-f effect upper bounds the useful signal strength. A-f effect in MEMS resonators occurs because of two different kinds of nonlinearities – electrical and mechanical. In the following subsections we analyze these mechanisms individually and drive analytical models based on 3rd order nonlinearity effects. We also examine the intermediate regime, where mechanical and electrical nonlinearities tend to cancel.

### A. Mechanical A-f Effect

Mechanical nonlinearities are similar to the kind seen in quartz [1], where the resonator response *bends* toward the higher frequency side. This is caused by mechanical stiffening of the effective spring constant at higher amplitudes. This effect is shown in Figure 2.


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[1]Current Address: Robert Bosch Corporation (Research and Technology Center), Palo Alto, California, USA

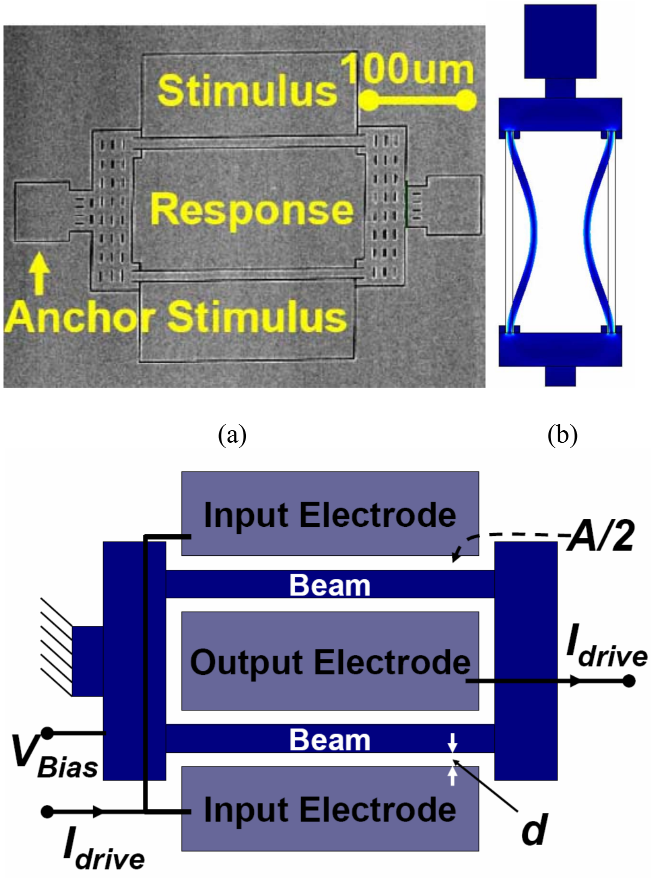


Figure 1. a) Scanning electron micrograph of the top view of the DETF resonator. The beam dimensions are L = 220 μm, w = 8 μm and t = 20 μm. The resonant frequency of this resonator is ~1.3 MHz, with a Q~10,000. b) FEMLAB simulation showing the resonant mode shape of the structure. c) connection diagram of the device.

We can express the mechanical nonlinearity in the system by the higher order terms in the mechanical restoring (or spring) force, which can be written as

$$F_{spring} \approx -k_1 x - k_3 x^3 \quad (1)$$

where $k_1$ is the linear spring constant, $k_3$ is the 3rd order force nonlinearity component and $x$ is the beam displacement. The 2nd order nonlinearity component has been ignored due to symmetry of the structure. The negative sign in (1) indicates the restoring nature of the force. For this system the mechanical *A-f* coefficient is given by [11, 17]

$$\kappa_m = \frac{3k_3 d^4}{4m\omega_0^4 \varepsilon^2 A^2 V_{Bias}^2} \quad (2)$$

where $d$ is the electrostatic gap size, $m$ is the effective mass of the structure, $\omega_0$ is the angular frequency, $A$ is the effective area of the transduction capacitance, ε is the permittivity of the material in the transduction gap and $V_{Bias}$ is the DC bias voltage on the structure. This effect may not be observed in devices with low $Q$ or very small gap size, as in these cases the system cannot be modeled with 3rd order nonlinearities alone. In this case, higher electrostatic nonlinearities cannot be neglected.

### B. Electrical A-f Effect

Electrical nonlinearities make the effective stiffness of the device smaller at high amplitudes. This causes the resonator response to progressively bend towards the lower frequency side as we increase the drive current. Figure 3 illustrates this effect. This can be modeled by looking at the higher order Taylor coefficients of the expression for the electrostatic force between the plates of a capacitor. Using the third order nonlinearity term, the electrical *A-f* coefficient becomes [11, 17]

$$\kappa_e = -\frac{3}{m\omega_0^4 \varepsilon A d} \quad (3)$$

In this regime, $\kappa$ is independent of the bias voltage. This effect should be observed at high bias voltages in almost every electrostatic MEMS resonator.

Note that the models presented up to this point are not specific to flexural designs, but are valid for all types of electrostatic resonators, with drive and sense symmetry such that 2nd order nonlinearities can be ignored.

### C. Intermediate A-f Effect

At intermediate voltages, an interaction electrical and mechanical of nonlinearities is observed. As amplitude is increased, the response shows the stiffening effect of mechanical nonlinearities followed by the softening effect of higher order electrical nonlinearities at higher amplitudes. Due to this inversion at these bias voltages, certain amplitudes of oscillation exist where the frequency dependence for small amplitude perturbations is negligible. Also, the critical bifurcation currents are higher at intermediate voltages. This effect is shown in Figure 4. The intermediate voltage is given by

$$V_{Bias} = \sqrt{\frac{k_3 d^5}{4\varepsilon A}} \quad (4)$$

## III. Experimental Results

For experimental verification we measure the frequency drift as we vary the drive current, for different values of the DC bias voltage, $V_{Bias}$. This is shown in Figure 5. At intermediate voltages there are certain amplitudes of oscillation at which the slope of the A-f curve becomes zero, and operating here would eliminate amplitude noise induced phase noise [12].

The extracted A-f coefficient for different values of $V_{Bias}$ is plotted in Figure 6. At intermediate bias voltages the A-f coefficient becomes very close to zero. In Figure 7 we plot the measured drive current at the onset of instabilities with respect to $V_{Bias}$.

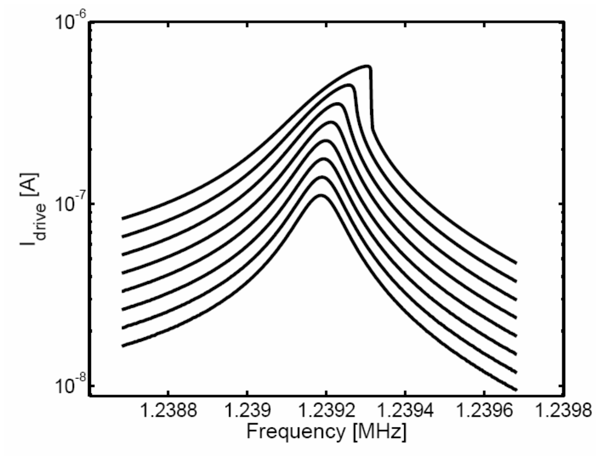


Figure 2. Measured mechanical A-f effect at $V_{Bias}$ = 25 V.

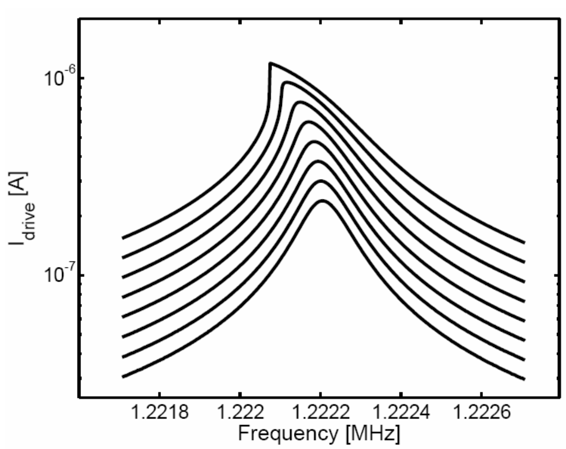


Figure 3. Measured electrical A-f effect at $V_{Bias}$ = 90 V.

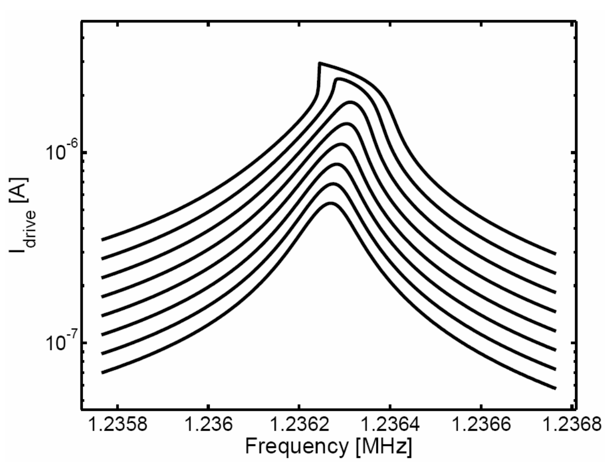


Figure 4. Measured intermediate A-f effect at $V_{Bias}$ = 44 V.

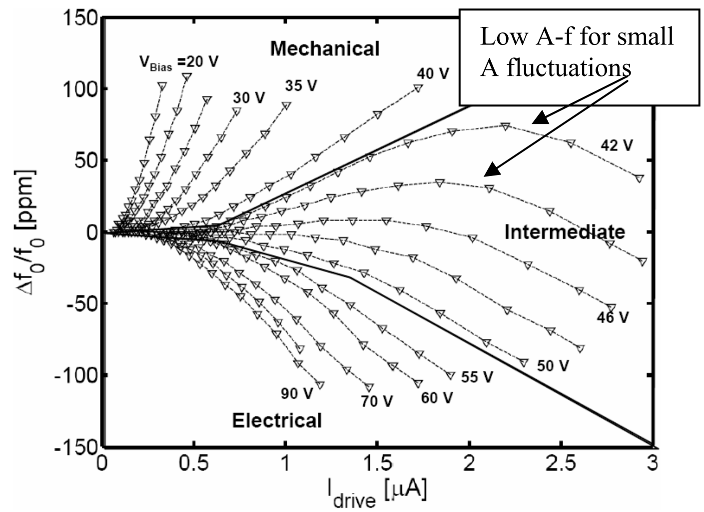


Figure 5. A-f effect in MEMS resonators. Frequency shift plotted with respect to the activity or drive current, at different bias voltages. The three regimes can be seen; mechanical, electrical and intermediate.

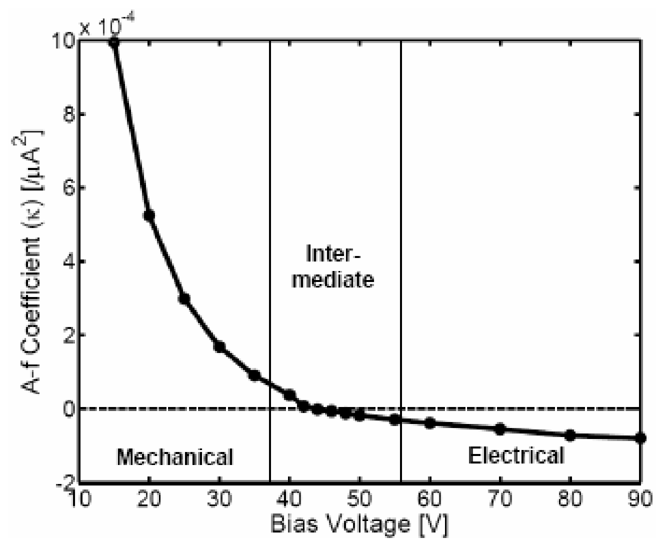


Figure 6. Extracted A-f coefficient from the frequency shift data of Figure 5.

## IV. DISCUSSION

Two of the main issues that are due to the A-f effect are amplitude fluctuation induced frequency fluctuation and Duffing bifurcation instabilities. The presented resonators show nonlinearity cancellation, which helps reduce both of these issues.

As can be seen from Figure 5, at intermediate bias voltages there are certain drive conditions at which the slope of frequency for small amplitude fluctuations is zero. Operating under these conditions will give better signal strength without suffering from amplitude-based frequency fluctuations.

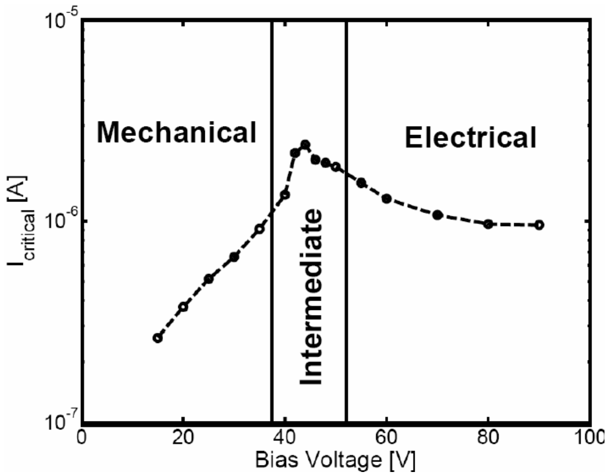


Figure 7. Drive current at the onset of critical duffing bifurcation plotted with respect to $V_{Bias}$.

In Figure 6, smaller values of the A-f coefficient κ are observed in the intermediate region. In this region, there is considerable but definite increase in the sustainable drive current, as can be seen in Figure 7. At higher current levels, higher order electrical nonlinearities eventually cause instabilities as evident from Figure 4.

Although the A-f coefficient is near zero for a certain $V_{Bias}$, higher order nonlinearities ultimately limit the resonator amplitude. The actual improvement in output current compared to the electrical regime is only about 3dB. In the electrical A-f regime, the A-f coefficient in our test structure is found to be about 9 orders of magnitude higher than in a 5 MHz AT cut quartz. However, from the derived model, we conjecture that this disadvantage will be less pronounced in resonators operating at higher frequency.

Several important design guidelines are obtained from the presented nonlinear modeling. The dependence of the A-f coefficient on *m* and *A* implies that a thicker device will make the resonator more linear, thereby making bulk micromachining fabrication processes superior to surface micromachining for these applications. Also, the expected improvement at higher frequency together with large mass and area suggested that arraying of resonators should lead to improved performance.

## V. Conclusions

In this work, we investigated the mechanisms and impact of nonlinearities on the frequency stability of MEMS resonators. Models for the A-f effect in MEMS resonators were developed and verified. Also, useful cancellation of nonlinearities was observed.

In our low frequency DETF test structure, we found that the A-f coefficient is several orders of magnitude larger (poorer) than quartz. However, we conjecture that A-f coefficient scales well with frequency and that higher frequency silicon resonators will have weaker A-f dependence.

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