Non-Linearity Cancellation in MEMS Resonators for Improved Power-Handling

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Abstract

In this work, we present mathematical analysis and experimental verification of the bifurcation limited power handling in MEMS resonators. We report useful cancellation between electrical and mechanical non-linearities. Within the scaling limits it has been found that the power handling improves for devices with larger electrode to resonator gaps. We also report an alternative method of measuring critical bifurcation using shifts in resonant frequency.

Introduction

Unlike quartz crystals, where the maximum power handling is limited by the sustaining electronics, the power handling in MEMS resonators is limited by device non-linearities. Having high power handling is very important for phase noise performance in any resonator based oscillator. Bifurcation of the resonator response in MEMS resonators due to mechanical and electrical non-linearities has been previously reported in [1] and [2]. Fig. 1 illustrates these effects. The maximum usable amplitude of oscillation has been reported in [2] and [3] to be the critical bifurcation amplitude. Beyond this point, the amplitude of oscillation depends on the prior conditions. In [4] and [5] it has been reported that the close to carrier phase noise is independent of amplitude below critical bifurcation condition and this noise is higher when the resonator is used in the bifurcation regime.

In this work we analytically model and measure the maximum output current before critical bifurcation and, as is often done in the case of quartz, identify this maximum output signal current as the power handling. The analytical models developed also provide a general and quantitative measurement method for detection of the onset of critical bifurcation by measuring the shift in resonant frequency due to non-linearities.

Theoretical Analysis

A. Non-linearity limited Power Handling

In this section we present theoretical analysis for calculating the maximum current output before critical bifurcation, in an electro-statically actuated and sensed MEMS resonator. Introducing stiffness non-linearities in the damped second order system equation, the new equation of motion becomes,

$$\ddot{x} + 2\lambda\dot{x} + \omega_0{}^2 x = \frac{f}{m}\cos\gamma t - \alpha x^2 - \beta x^3 \qquad (1)$$

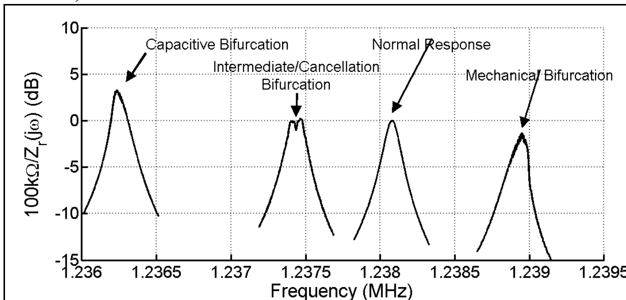

Fig 1. Measured resonator response showing different kinds of Bifurcation, taken at different drive and bias voltages. The difference in resonant frequency on different curves is a result of spring softening due to different Bias Voltages. Z_r is the single port resonator small signal impedance.

It is also shown in [6] that the eigenfrequency of the above system under the assumption of small oscillations will be given by,

$$\omega_i = \omega_0 + \kappa x^2 \qquad (2)$$

Where κ is given by,

$$\kappa = \left(\frac{3\beta}{8\omega_0} - \frac{5\alpha^2}{12\omega_0{}^3} \right), \qquad (3)$$

And the critical force for bifurcation is given by,

$$f_k{}^2 = \frac{32 m^2 \omega_0{}^2 \lambda^3}{3\sqrt{3}\,|\kappa|} \qquad (4)$$

For an electro-statically actuated resonator the actuation force is given by,

$$f = \frac{\varepsilon A}{d^2} V_{Bias} V_{AC} \le f_k \quad . \qquad (5)$$

The inequality expresses the condition on this actuation force to avoid bifurcation.

TABLE I
LIST OF SYMBOLS USED

Symbol	Description
λ	Damping Coefficient. Also equal to half of the -3dB bandwidth.
ω_0	Natural Frequency.
γ	Excitation frequency
α	2nd order non-linearity coefficient.
β	3rd order non-linearity coefficient
ω_i	Eigenfrequency under the influence of non-linearities.
m	Effective mass of the resonator.
d	Electrode to resonator gap size.
V_{Bias}	DC bias voltage.
V_{AC}	Input Ac drive voltage.
f	Driving force on the resonant structure.
f_k	Critical driving force for bifurcation.

The output current of the resonator in terms of the actuation force is given by,

$$I_{out} = \frac{Q}{\sqrt{km}}\frac{\varepsilon A}{d^2}V_{Bias}f = \frac{1}{2m\lambda}\frac{\varepsilon A}{d^2}V_{Bias}f \qquad (6)$$

By combining equations (4) and (6) we can get the maximum output current before bifurcation as,

$$I_{out,max} = \frac{\varepsilon A}{d^2}V_{Bias}\omega_0\lambda^{1/2}\sqrt{\frac{8}{3\sqrt{3}\,|\kappa|}} \qquad (7)$$

We can now use equation (7) to calculate the maximum output current limited by the onset of bifurcation for different cases. Two types of non-linearities are observed in these systems; mechanical and electrical. In this equation only κ depends on the non-linearities in the system. We can find out κ for each case and find the maximum output current accordingly.

Leeson's Result – It can be seen from the dependence of maximum output current on damping (λ) that the $1/f^2$ noise becomes independent of the quality factor Q ($\propto \lambda^{-1}$), because the P_{sig} depends on $1/Q^2$. The noise floor (far from carrier noise) improves for a lower Q device, because of higher signal power possible for Low Q devices. Hence aiming for very high Q may become suboptimal from the perspective of phase noise in electrostatic MEMS resonators in which power handling is limited by bifurcation.

Resonant Frequency Shift – The resonant frequency depends on the amplitude of vibration as given in eq. (2). It has been shown in [6] that when critical bifurcation occurs, it occurs at a frequency offset of $\sqrt{3}\lambda$ from the natural frequency ω_0. It has also been mathematically shown that the offset in the eigenfrequency under these drive conditions is 2/3 of the bifurcation offset. Hence, offset in eigenfrequency is given by,

$$\omega_i - \omega_0 = \kappa b^2 = \frac{2}{\sqrt{3}}\lambda = \frac{\Delta\omega_{-3dB}}{\sqrt{3}}. \qquad (8)$$

It is important to note here that ω_0 includes the "linear" effects of DC bias voltage, called spring softening. Hence this frequency shift will be the difference in resonant frequency at low input AC drive, when there is no effect of non-linearities, and the resonant frequency at critical bifurcation caused by changing the Ac drive and not the bias voltage.

B. Mechanical Bifurcation

The force-displacement relation for mechanical non-linearities for a flexural beam case, which is symmetric for positive and negative displacement, can be approximated by,

$$f = k_1x + k_3x^3 = k_1x + m\beta_m x^3 \qquad (9)$$

Hence, it follows that

$$\kappa_m = \frac{3\beta_m}{8\omega_0} = \frac{3k_3}{8m\omega_0} \qquad (10)$$

Substituting (10) into (7) we get,

$$I_{out,max} = 2.56\frac{\varepsilon A\omega_0^{3/2}m^{1/2}}{k_3^{1/2}}\frac{V_{Bias}\lambda^{1/2}}{d^2} \qquad (11)$$

If k_3 is positive, it means that the spring becomes stiffer as the amplitude increases and this would cause bifurcation in which the resonator response bends towards the right side as shown in figure 1.

C. Electrical Bifurcation

Electrical Bifurcation is more complex because of the possible presence of both 2nd order and 3rd order non-linear effects. In some devices like the flexural beam structure shown in figure 2, the 2nd term may get cancelled due to symmetry (or be present due to manufacturing tolerances). In other structures such as breathe mode structures such cancellation may not happen, depending on the configuration of input and output electrodes. In most cases the structure can be accurately modeled only as an intermediate combination of both these effects due to process tolerances etc. In order to get a clearer dependence of maximum output current on design parameters such as gap size (d) and bias voltage (V_{Bias}), we will separate these into the extreme cases namely, the 2nd order dominated and 3rd order dominated extremes.

2nd Order Electrical Non-Linearity Dominated – As has been shown in [2], the 2nd order capacitive or electrical non-linearity term can be obtained as,

$$\alpha_e = -\frac{3V_{Bias}^2\varepsilon A}{4md^4} \qquad (12)$$

Using this result we can obtain the maximum output current when limited by 2nd order capacitive non-linearities by substituting (12) in (3) and then into (7) as,

$$I_{out,max} = 1.95m\omega_0^{5/2}\frac{\lambda^{1/2}d^2}{V_{Bias}} \qquad (13)$$

This shows that the maximum output current decreases increasing the bias voltage and increases with increasing gap size. While this may seem counter-intuitive, it should be noted that the input stimulus to these devices is not fixed but depends on the onset of critical bifurcation. Here although the motional impedance decreases, as expected, if the bias voltage is increased and the gap size is decreased, doing this increases the capacitive non-linearities enough that the maximum output current before occurrence of bifurcation decreases.

3rd Order Electrical Non-Linearity Dominated – Similarly, the 3rd order capacitive or electrical non-linearity term can be obtained as,

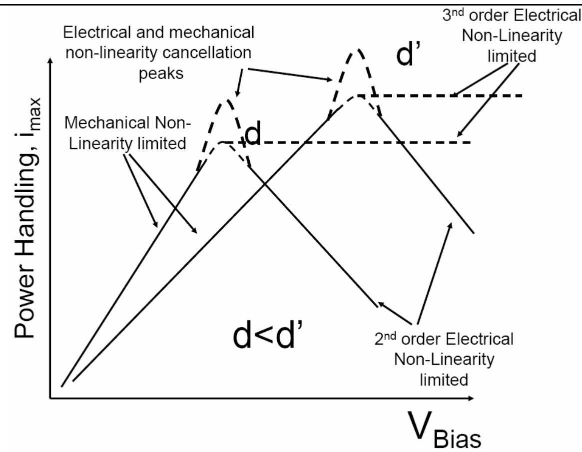

Figure 2. Theoretical dependence of power handling on Bias Voltage and gap size ***d*** for a generic electrostatically actuated Si MEMS resonator. It can be noted that the dependence is different in the electrical bifurcation regime when the dominant electrical non-linearity terms are 2nd order and when they are 3rd order.

$$\beta_e = -\frac{2V_{Bias}^2 \varepsilon A}{3md^5} \tag{14}$$

and the corresponding maximum output current is,

$$I_{out,max} = 3.51\left(m\omega_0^{\ 3}\varepsilon A\right)^{1/2}\left(\lambda d\right)^{1/2} \tag{15}$$

Here again we see that the output current increases with increasing gap size, in spite of decreasing the motional impedance. Eqs. (13) & (15) tell us that getting better power handling and hence better phase noise in MEMS resonator based oscillators is in departure from reducing the motional impedance alone.

D. Non-Linearity Cancellation

As has been reported in [2], in most cases of Silicon MEMS resonators the mechanical stiffness non-linearities work towards making the structure stiffer and thereby cause "right hand side bifurcation" as shown in Fig. 1. Capacitive non-linearities occur at higher bias voltages when the capacitive forces are stronger and cause "left hand side bifurcation," also shown in Fig. 1. There exists an intermediate regime, where the capacitive non-linearities cancel out the mechanical non-linearities thereby allowing higher amplitude of oscillation and hence higher output current. This is schematically explained in Fig. 2.

Experimental Results

We used a 1.3MHz Doubled-Ended Double Anchored Tuning fork resonator (DETF), shown in figure 3, fabricated using the epi-seal encapsulation process as discussed in [7] for these experiments. The experimental setup used for measuring maximum output current is shown in Fig 4.

The measured maximum output current, measured on two different samples, before critical bifurcation is shown in Fig. 5. For making this measurement the input AC voltage to the

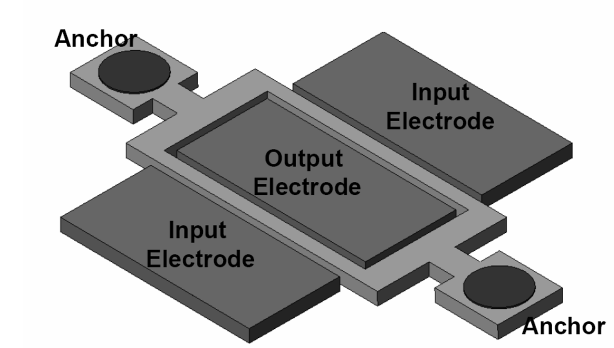

Figure 3. Schematic of the DETF resonator used. The drawn gap size is 1μm, the beam lengths are 200μm with a beam width of 8 μm. These devices were defined on an SOI wafer and then encapsulated with an epi-polysilicon cap.

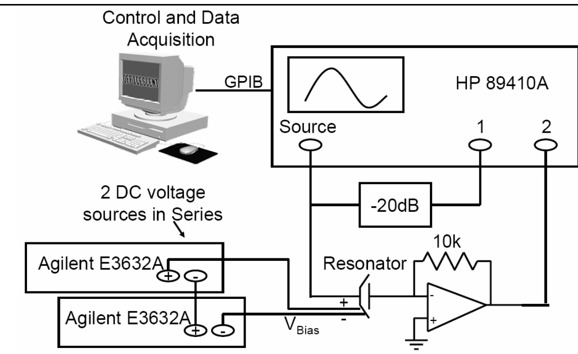

Figure 4. Diagram illustrating the measurement setup used for measuring the power handling. The resonator output current was amplified using a single transimpedance stage of gain 10kΩ. The 20dB attenuator helps prevent overloading of the signal analyzer input channel.

resonator was increased in increments of 1dB, until bifurcation is observed. The drive conditions 1dB below this bifurcation point was used as the max output current before critical bifurcation, and accordingly error bars corresponding to 1dB error bars have been inserted on the presented data.

The frequency shift at critical bifurcation was also measured and is shown in Figure 6. Figure 7 shows measured phase noise of an Oscillator based on this resonator using compression gain control.

Conclusions

It is evident from Fig. 5 that cancellation between electrical and mechanical non-linearities can be achieved, by selecting the correct gap size and correct bias voltage.

It can also be seen from (13) & (15) that the power handling increases with increased gap size in the electrical bifurcation dominated regime. Due to cancellation between the two kinds of non-linearities, the overall maximum output current possible will be larger for devices with larger gaps, if high bias voltage can be applied. There is a trade-off here that with larger gaps, larger bias voltages will be needed.

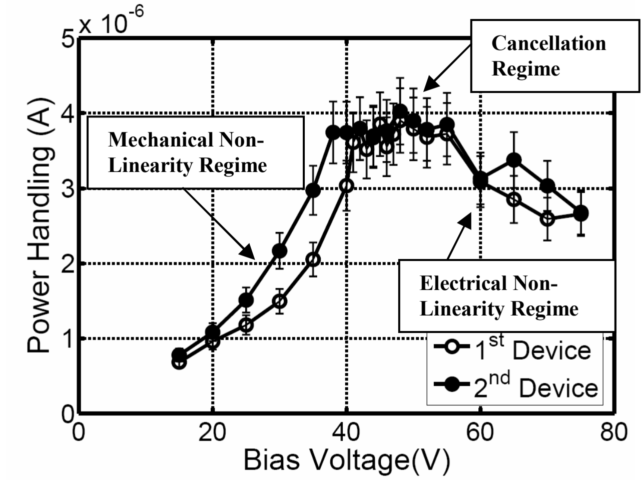

Figure 5: Measured output maximum current at the onset of bifurcation at each bias voltage for two different devices. The two devices are the same design fabricated on the same silicon SOI wafer. This measurement was taken by increasing the input stimulation level for each bias voltage and then noting the output current onto a 0Ω load (Fig 4) at the onset of bifurcation.

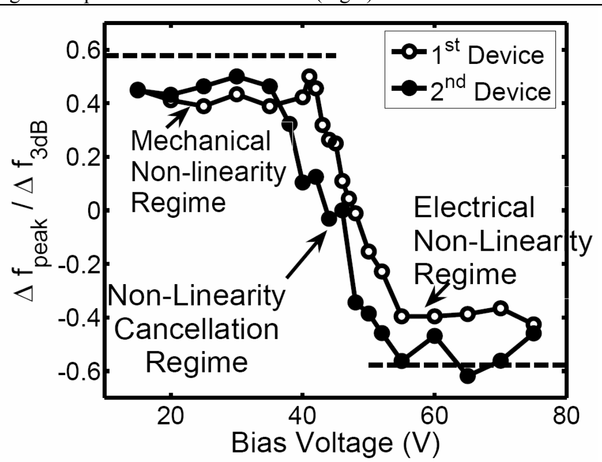

Figure 6: This shows the ratio of peak frequency shift (between the perfectly linear resonant frequency which occurs at low excitation and the resonant frequency when a device at critical duffing, at the same DC bias voltage) and the -3dB bandwidth of the device. The ratio changes sign as the non-linearities go from mechanical to electrical, but remain close to the predicted value of $\pm 1/\sqrt{3}$ (drawn as dashed lines for reference) except in the intermediate or cancellation regime. This data produces points towards the possibility of the non-linearity induced frequency shift as a method to detect critical bifurcation on devices in oscillator circuits, where only frequency is a measurable parameter.

From Fig. 6, we get an alternative and quantitative method of detecting the critical bifurcation point using the non-linearity induced frequency shifts. This is particularly useful for detecting critical bifurcation in oscillators where the resonator response measurement is unavailable.

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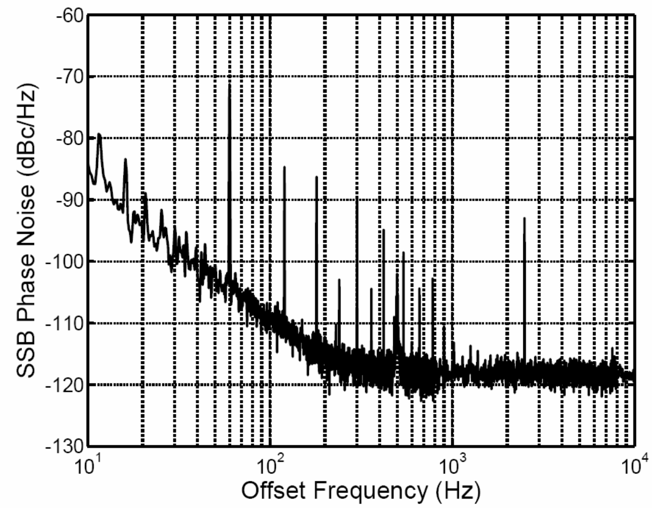

Figure 7: Measured Single Side Band phase noise using a compression based gain control oscillator with an input drive level of ±150mV, i.e., about 6dB below critical bifurcation. These measurements were done using a PSI ODIN-320A phase noise measurement instrument.

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