EFFECTS OF MECHANICAL VIBRATIONS AND BIAS VOLTAGE NOISE ON PHASE NOISE OF MEMS RESONATOR BASED OSCILLATORS

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ABSTRACT

Micromechanical Resonator based oscillators are a promising technology for replacing quartz crystal based oscillators. In this work, we will report the effects of mechanical vibrations and bias voltage noise on the phase noise performance of electrostatic MEMS resonator based oscillators. Accurate models for both these effects are discussed along with their experimental verification using a 1.3MHz, epi-silicon encapsulated Single Anchored Double Ended Tuning Fork (DETF) resonator. The acceleration sensitivity of the resonator was found to be <10ppb/g which is better than many low cost crystal resonators, and shows potential for improvement to get performance which is at par or better than quartz crystal oscillators.

1. INTRODUCTION

Remarkable progress has been made over the past several years on understanding and designing MEMS resonators in based oscillators for various timing or frequency reference applications. Several groups have characterized MEMS resonators for low noise oscillator applications. Notably, MEMS resonator based oscillators with phase noise that satisfies GSM requirements has been reported in [1]. Phase noise improvement by making mechanically coupled resonator arrays has been demonstrated in [2]. Most of these works improve phase noise by designing robust devices with large power handling (signal power available, to improve the signal-to-noise-ratio) and reducing the electronics noise.

These results ignore 2nd order effects such as those due to external mechanical vibrations and noise in the DC bias supply. In most real world applications, like navigation systems, cell phones and mobile radars, creating a vibration free environment is almost impossible. Due to this vibration sensitivity is a very important specification even for quartz crystal oscillators. In applications such as CMOS integrated MEMS or system-on-chip, generating an ultra-clean DC bias supply may be an issue as big capacitor banks may be required for bypassing noise. Since quartz crystal resonators do not need a DC biasing supply for operation, this problem is specific to electrostatic MEMS resonators. If these issues are not considered properly during system/device design, these two noise sources may be the ones limiting the phase noise performance rather than the electronics noise.

A review of vibration sensitivity in crystal oscillators is presented in [3]. Acceleration induced frequency shifts,

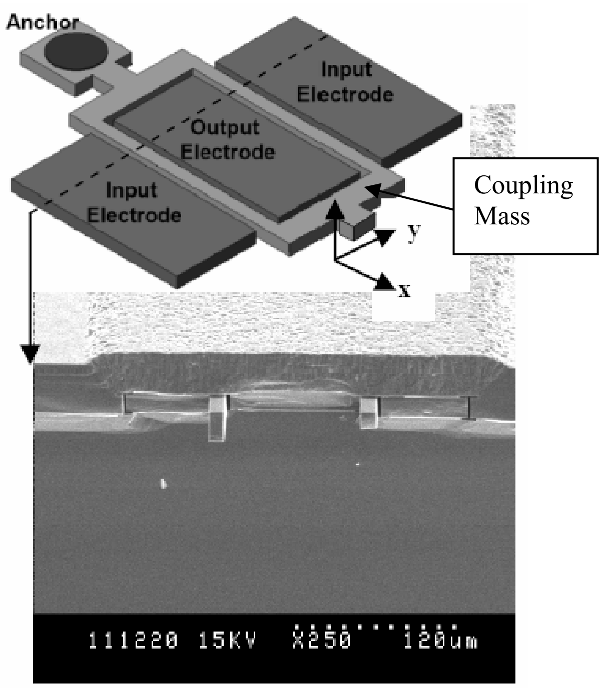


Fig 1. Schematic Diagram of 1.3MHz Single Anchored, Double Ended Tuning Fork (DETF) resonator device and cross sectional SEM.

which cause the phase noise due to acceleration, in MEMS oscillators have been reported in [4] and [5] which deal with resonant accelerometers. Phase noise due to noise in the bias voltage (or equivalently in the ground plane or input of sustaining amplifier) has previously been discussed in [6].

In this work we investigate the noise due to these effects in a single anchored double ended tuning fork (DETF) device. This device is fabricated using an epi-seal encapsulation process as described in [7]. The frequency of this device is 1.3MHz, fabricated on a <100> SOI wafer with beams oriented in the [110] direction. The beam dimensions are 220µm (*l*) X 8µm (*w*, along the direction of oscillation) X 20µm(*h*). A schematic and SEM of the device are shown in figure 1. The coupling block used for coupling the two beams together and keeping the beams acting as double clamped had a total size of 135µm (*l*) X 40µm(*w*) X 20µm(*h*).

2. THEORETICAL ANALYSIS

In this section we provide analytical models for vibration and bias voltage sensitivity in MEMS resonator based oscillators.

Vibration Sensitivity

As has been shown in [3] dependence of resonant frequency on acceleration is given by,

$$f_r\left(\vec{a}\right) = f_0\left(1 + \vec{\Gamma}\cdot\vec{a}\right) \qquad (1)$$

where Γ is the acceleration sensitivity and a is acceleration. In an oscillator circuit such frequency dependence will cause the accelerations to cause frequency modulation, which will give a single sideband phase noise level (in dBc, or dB with respect to carrier) for low modulation condition ($f_V/f_0<0.1$) –

$$L\left(f_0 \pm f_V\right) = 20\log\left(\frac{\left(\vec{\Gamma}\bullet\vec{a}\right)f_0}{2f_V}\right) \qquad (2)$$

Here L is the side band noise level at a frequency offset of f_V from the carrier. This noise would appear as $\boldsymbol{1/f^2}$ noise if the oscillator is under "white" vibration noise. Please note that this equation will give the phase (random) noise level in *dBc/Hz* if a is random vibration noise power in $g/\sqrt{Hz}$. In our case of a DETF, single anchored device this vibration sensitivity is caused by the axial stress induced due to external acceleration. From [8] we can use the axial stress sensitivity of resonant frequency and get the acceleration sensitivity as -

$$\Gamma_x = \left(\frac{\partial\left(f_r/f_0\right)}{\partial\sigma_x}\right)\left(\frac{\partial\sigma_x}{\partial a_x}\right) = \left(0.121\frac{12L^2}{\pi^2 E w^2}\right)\left(\frac{m}{2hw}\right) \qquad (3)$$

Where $\boldsymbol{L}$, $\boldsymbol{w}$ and $\boldsymbol{h}$ are the length, width of the DETF and $\boldsymbol{m}$ is the effective mass which causes axial stress in the beams. It can be shown analytically as well as with FEM that the y and z direction sensitivities are negligible for the y-direction resonant frequency. Ignoring the effect of the beams in calculating the axial stress and using just the coupling mass (135μm X 37μm X 20μm) we get a sensitivity of ***4.92 ppb/g***. Since x-direction is aligned with the <110> crystal direction we use ***E=168GPa***. However the beams are a distributed load which causes axial stress. We can accurately model its effect by using FEMLAB and we get a simulated x sensitivity of $\boldsymbol{\Gamma_x=7.37}$ ***ppb/g***, as shown in Figure 2.

Bias Voltage Sensitivity

The resonant frequency dependence on bias voltage due to spring softening [6] can be expressed as,

$$f_r \approx f_0\left(1 - \frac{CV_{Bias}^2}{kd^2}\right) \qquad (4)$$

where $\boldsymbol{k}$ is the mechanical stiffness of the resonator, $\boldsymbol{d}$ is the electrode to beam gap and $\boldsymbol{C}$ is the total input (or output) electrode to beam capacitance. This model predicts that lower bias voltages and bigger gaps would improve the phase noise due to bias voltage noise. Figure 3 shows measured dependence of resonant frequency on the bias voltage.

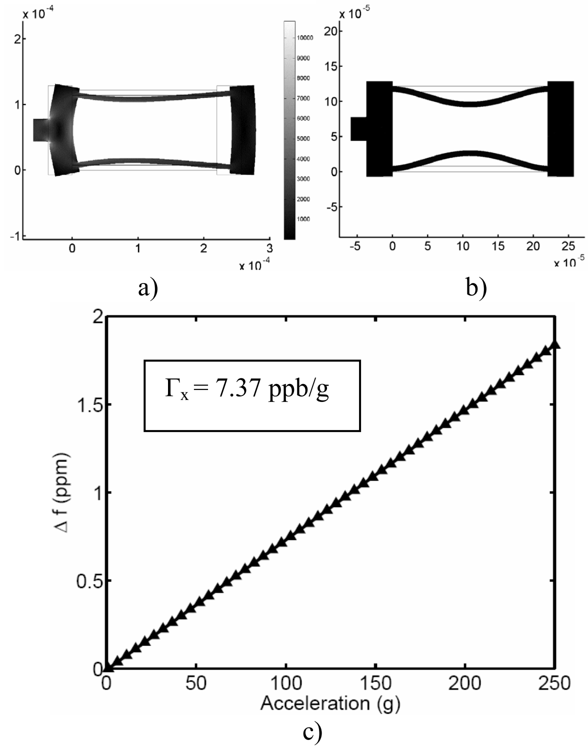


Figure 2. FEMLAB simulations for obtaining the vibration sensitivity in a MEMS DETF resonator. a) Static deformation under the influence of acceleration, b) the resonant mode of the DETF obtained using the initial stress conditions obtained from (a). c) plot showing the change in resonant frequency as a function of the applied acceleration.

For small changes in bias voltages such as the noise in bias voltage supply, the dynamic frequency change is given by,

$$f_0\left(\Delta V_{Bias}\right) = f_0\left(1 - \alpha\cdot\Delta V_{Bias}\right) \qquad (5)$$

where α is a linearized bias voltage sensitivity factor which is directly proportional to V_{Bias}, obtained by differentiating equation (4) w.r.t. V_{Bias}.

It can be seen that equation (5) is mathematically similar to (1) and hence a phase noise due to bias voltage noise is given by,

$$L\left(f_0 \pm f_{Noise}\right) = 20\log\left(\frac{\alpha v_{AC} f_0}{2f_{Noise}}\right) \qquad (6)$$

Here $\boldsymbol{v_{AC}}$ is the noise power on the bias voltage supply at a frequency f_{Noise}. It is observed that increasing bias voltage helps in reducing the noise floor (or far from carrier phase noise) by increasing the signal output from the resonator as compared to the electronics noise, but it increases the $\boldsymbol{1/f^2}$ noise of the oscillator due to the noise in the DC bias voltage supply, which indicates a tradeoff if bias voltage dependent phase noise is dominant at low offset frequencies. Also it can be noted from equation (6) that ***1/f*** noise on the DC bias supply gets converted to $\boldsymbol{1/f^3}$ phase noise in oscillators.

Apart from this, the output current from the resonator is given by,

$$I_{out} \propto V_{Bias}^2 \cdot V_{Input} \qquad (7)$$

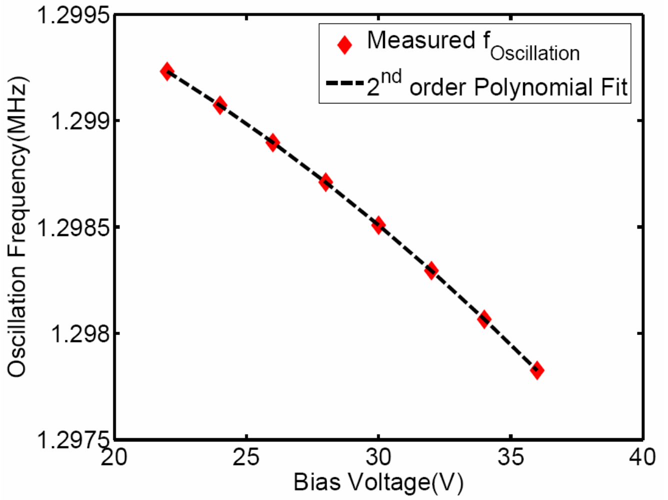


Figure 3: Shows the resonant frequency variation with Bias voltage. As predicted by analytical models, this frequency dependence can be accurately described by a 2^{nd} order dependence on bias voltage. The slope of this curve at any bias voltage gives the resonant frequency sensitivity to small perturbations (noise) in the bias voltage supply (α). At 35V, $\alpha = 93 ppm / Volt$ from this plot.

noise in bias voltage would hence cause a single side-band amplitude noise due to noise multiplication given by,

$$L\left(f_0 \pm f_{Noise}\right) = 20\log\left(\frac{v_{AC}}{V_{Bias}}\right) \quad (8)$$

3. EXPERIMENTAL RESULTS

Vibration Sensitivity Results

Figure 4 shows the experimental setup used for measuring the vibration sensitivity of our MEMS resonator device. The oscillator PCB contains the resonator with a customized sustaining transimpedance amplifier and compression based gain control. Figure 5 shows a typical comparison of the spectrum analyzer output with and without mechanical vibrations.

Figure 6 shows the x-direction vibration sensitivity extracted from the side-peak level using the expression in equation (2), plotted for different values of bias voltages. The vibration sensitivities of low cost AT crystals and precision SC cut crystals have also been shown on this figure for comparison. The vibrations sensitivities in the y and z directions were found to be immeasurably low in confirmation with our model. The measured x-sensitivity shows good match with the simulated vibration sensitivity.

Bias Voltage Sensitivity results

The bias voltage sensitivity was measured using the experimental setup illustrated in figure 7, by injecting signals with different noise frequencies through the 33120 function generator and recording the side-peak level observed. The output spectrum shows side-peaks at the an offset equal to the noise frequency from the resonant frequency similar to those shown in figure 5. The extracted bias voltage frequency

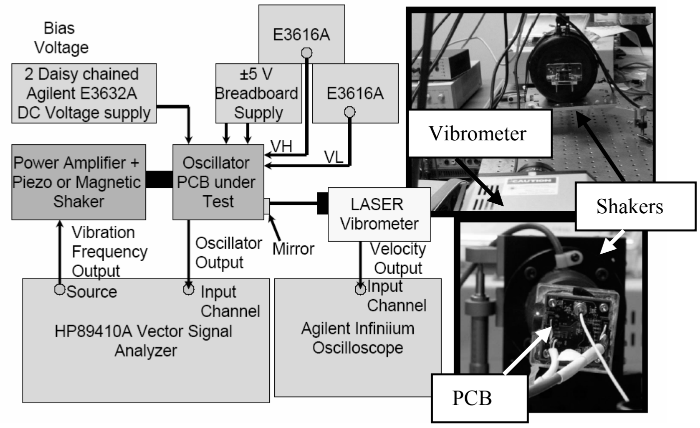
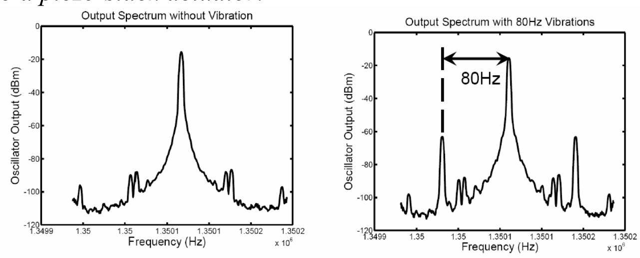


Figure 4. Schematic presentation of the Experimental setup used for the vibration tests, along with photographs. Among the two pictures on the right, the top shows the laser vibrometer with its beam focusing onto the PCB attached to a magnetic shaker. The bottom picture shows the PCB attached to a piezo-stack actuator.



a) b)

Figure 5. Spectrum of oscillator output, under the influence of 80Hz vibrations. The vibration sidebands can be observed at 80 Hz offset from the carrier or resonant frequency. a) shows the output spectrum without any external vibration, b) output spectrum with 80 Hz external vibrations.

sensitivity is shown figure 8. It was observed that the side peak levels on the left and right hand sides were different, this can be mathematically attributed to the cancellation between the amplitude and phase noise on the higher frequency (right) side. The figure also shows *93ppm/Volt*, which is the predicted vibration sensitivity from frequency shift measurements as shown in figure 3.

4. CONCLUSIONS

It is evident from equation (3) MEMS resonators with vibration sensitivities comparable to good quartz crystals can be designed. However other mechanisms such as spring softening, as noted in [4] also need further investigation.

It can be seen from equation (4) that smaller gap sizes (nano-gaps) and high bias voltage can make the MEMS resonator more prone to bias voltage noise, which is at a tradeoff with the far-from-carrier-phase noise requirement which may require higher bias voltage to be employed to increase the output current of the resonator. Equation (6) suggests that white noise in the bias voltage supply will be converted to $1/f^2$ phase noise in MEMS oscillator and $1/f$ noise in the supply will be converted to $1/f^3$ phase noise.

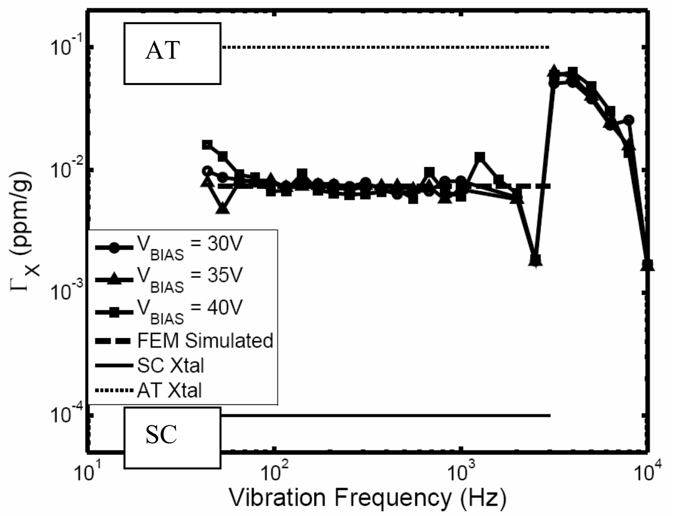


Figure 6: Vibration Sensitivity in X direction (vibration along the length of the beams) for different bias voltages. The measurements with f_v<1kHz were conducted with the magnetic shaker and the measurements with f_v>1kHz used the piezo-stack. The value of sensitivity remains independent of Bias voltage as can be seen here. FEM simulated value for the sensitivity is also shown. For comparison, acceleration sensitivities of typical Xtal Oscillators are also plotted here. The peaking in the sensitivity at high frequency is probably due to mount resonances in the measurement setup.

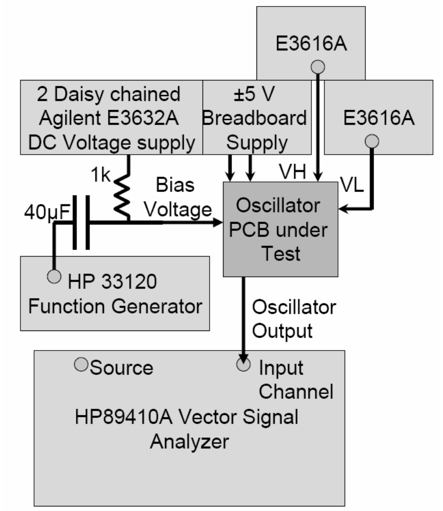


Figure 7: Schematic presentation of the setup used for measuring the effect of Bias Voltage noise. The 33120 function generator is being used here as the "noise" source.

5. ACKNOWLEDGEMENTS

This work was supported by DARPA HERMIT (ONR N66001-03-1-8942), Robert Bosch Corporation (RTC) and the National Nanofabrication Users Network facilities funded by the National Science Foundation under award ECS-9731294, and The National Science Foundation Instrumentation for Materials Research Program (DMR 9504099).

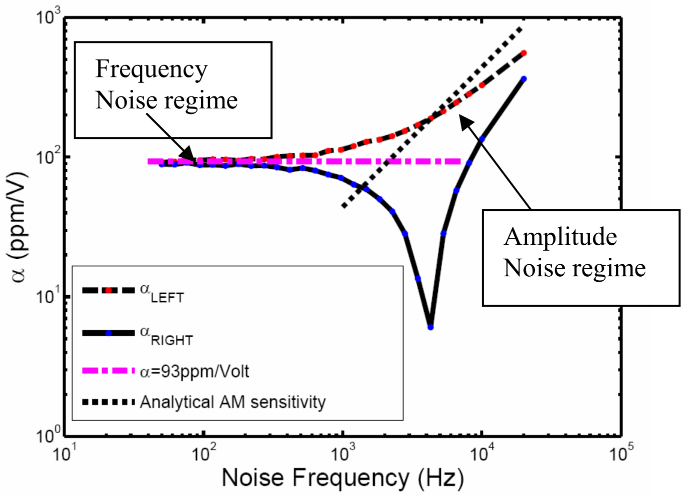


Figure 8: Bias voltage sensitivity α at 35V bias, plotted vs. noise frequency. The higher frequency side-peak (Right) and lower side-peaks (left) show different sensitivities and hence both have been plotted here. The dip in the α_{RIGHT} is caused by cancellation of Amplitude and Frequency noise induced side-peaks. This causes disparity between the levels of the left and right hand side-peaks and hence difference in sensitivities. The expected α in the Frequency noise regime, from spring softening measurements is shown and so is the analytically calculated sensitivity in the AM regime.

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