IMPACT OF SLOT LOCATION ON THERMOELASTIC DISSIPATION IN MICROMECHANICAL RESONATORS

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ABSTRACT

The effect of thermoelastic dissipation as an energy loss mechanism in micromechanical resonators is described. Specifically, we demonstrate that slots in resonant beams disrupt the heat flow from thermoelastic dissipation and alter the quality factor. Fully coupled finite element solutions and experimental verification are used to show that the *location* of the slots micromachined into the beams have a strong impact on the quality factor. Slots machined near the anchors and near the center of the beams are shown to have the strongest influence on quality factor. This is an indication of the complex interaction between the thermal and mechanical domains.

Keywords: Resonator, Thermoelastic Dissipation, Quality factor

INTRODUCTION

Micromechanical silicon resonators are considered by many to be on the verge of making a major impact as alternatives to quartz oscillators for frequency reference applications [1]. However, a solid understanding of how these resonators operate is imperative if they are to displace a well established technology such as quartz. One of the major areas of micromechanical silicon resonators which is still not entirely understood is quality factor, Q. Quality factor is a complex parameter, because it is influenced by all the different ways a resonator can lose energy. This includes gas damping, anchor loss, intrinsic material loss, surface dissipation, and thermoelastic dissipation (TED). This work is an investigation of TED as an energy loss mechanism in silicon micromechanical resonators.

BACKGROUND

Clarence Zener first formalized thermoelastic dissipation in 1937 [2, 3]. Theoretical models of TED have since been reformulated and expanded several times [4-6]. Recent work by others has treated thermoelastic dissipation with a modified form of Zener's theory by applying it multiple times to different geometric features within the resonator [7, 8]. However, this method is not generally accurate, and a more advanced treatment is required. The finite element method enables more advanced treatments, and is beginning to see use for prediction of quality factor in resonators [9]. This work utilizes finite element simulations that are derived from the fundamental equations of thermoelasticity [10, 11].

The process of thermoelastic dissipation can be described with the following example. A beam is flexed, placing one side in tension and the other in compression, fig. 1. The side in compression will get slightly warmer, and the side in tension will get slightly cooler due to the coupled nature of the mechanical and thermal domains. A temperature gradient now exists across the beam. A thermal gradient across a material with nonzero thermal conductivity will result in heat flow. This heat flow is an irrecoverable energy loss, a Q limiting mechanism. This is because mechanical energy is used to generate the temperature gradient. If the temperature gradient relaxes, this energy can not be returned to the mechanical domain. Another way to view this energy loss is that energy is used to change the entropy of the system, and this energy is permanently lost to the entropy upon relaxation of the temperature gradient.

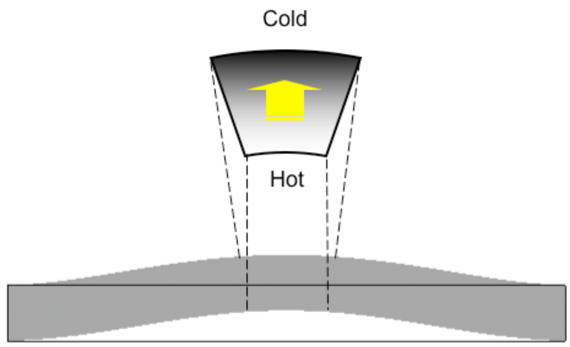

Figure 1. Conceptual drawing of process of thermoelastic dissipation on a flexed beam. Differential strain leads to temperature gradient, which leads to heat flow.

Zener developed theory for the 1-dimensional case of thermoelastic dissipation. In doing so, he defined a thermal time constant, τ, which is representative of the time necessary for a temperature gradient to relax, equation 1. It comes from the solution for the fundamental thermal mode of the beam.

$$\tau = \left(\frac{b}{\pi}\right)^2 \frac{C_p \rho}{\kappa} \qquad (1)$$

b is the dimension across the beam in the direction of the flexing. C_p is the specific heat capacity at constant pressure. ρ is the material density, and κ is the thermal conductivity. Zener compared this thermal time constant to the mechanical frequency of vibration, ω, to predict the maximum possible Q as limited by TED, equation 2. Of course, the Q could be lower if an energy loss mechanism other than TED dominated, but the Q could never be any higher than the value predicted by the equation.

$$Q_{TED} = \left(\frac{C_p \rho}{E \alpha^2 T_0} \right) \frac{1 + (\omega\tau)^2}{\omega\tau} \qquad (2)$$

The comparison between thermal time constant, τ, and mechanical frequency of the beam, ω, results in a Lorentzian-type function, There is also a coefficient to the Lorentzian which depends on specific heat capacity, density, Young's Modulus (E), thermal coefficient of expansion (α), and temperature (T_0). The Lorentzian gives three regions of behavior for the Q-limiting equation, figure 2. At high frequency, there is no heat flow because the beam flexes back and forth faster than the heat can flow. A temperature gradient is formed across the beam, but the beam flexes back before the temperature gradient can relax. Therefore, no energy is lost. This frequency range is typically referred to as the adiabatic regime In the middle range of frequencies, where the energy loss is the greatest, the beam flexes and allows just enough time for the temperature gradient to relax before the beam changes direction. At low frequency, no temperature gradient is formed, because the temperature gradient is dependent on the strain rate. This frequency range is typically referred to as the isothermal regime.

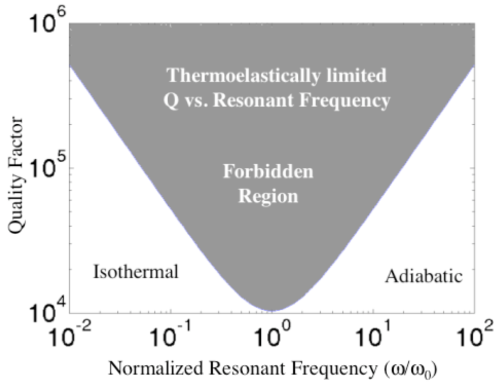

Figure 2. Limitation of Q from thermoelastic dissipation using Zener theory.

Zener theory works quite well for simple beams. However, it is not suitable for structures with geometry more complex than a simple beam. This is because only a single thermal time constant couples well into the mechanical resonant mode of a simple beam, while a more complex beam will be affected by many thermal time constants. The finite element method is necessary if TED-limited Q for arbitrary geometries is to be solved. One way to use the finite element method to solve this problem is to recreate Zener's method. Zener's method involved three steps (1) Solve for the desired mechanical resonance without taking the thermal domain into acocount. (2) Solve for the thermal modes, including the mechanical resonance. (3) Calculate the interaction, or "overlap", between the mechanical and thermal modes. This overlap determines how the mechanical energy is lost to the thermal domain, which determines Q. Another finite element method for determining Q is to use the fully-coupled thermal-mechanical equations [10, 11]. This avoids explicit calculation of the individual thermal and mechanical modes and solves the entire system simulatenously. This work uses the fully-coupled simulations. Equations 3 and 4, which are essentially a wave equation and Fourier's Law with additional coupling terms, are the basis for the fully-coupled simulations. Displacement is represented by u, and Poisson's ratio is ν.

$$\rho \frac{\partial^2 u}{\partial t^2} = E \frac{\partial^2 u}{\partial x^2} + \underbrace{\frac{\alpha E}{1-2\nu} \frac{\partial T}{\partial x}}_{coupling\ term} \qquad (3)$$

$$C_p \rho \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial t^2} - \underbrace{\frac{\alpha \mathrm{E} T_0}{1-2\nu} \frac{\partial}{\partial x} \frac{\partial u}{\partial t}}_{coupling\ term} \qquad (4)$$

RESULTS

It has been previously shown that micromachining slots into flexural resonant beams can have a significant impact on the Q of the resonator [12]. While the previous work verified that slots did have an impact on the resonator Q, it did not investigate if there is any dependence of Q on *where* on the beam the slot was located. This work provides insight about where on the beams slots should be placed to have the greatest impact.

Some insight into where slots may have an effect can be seen from the fully-coupled finite element simulation of a simple beam. As can be seen in fig. 3, regions where the strain gradient is more pronounced are likely to generate a larger temperature gradient.

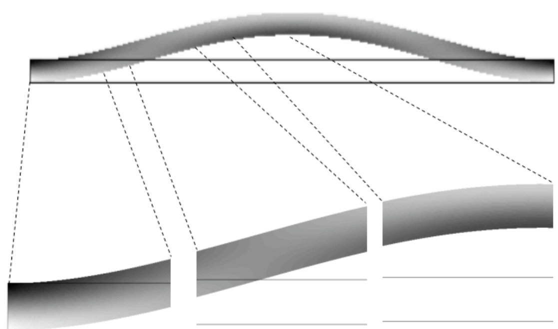

Figure 3. Temperature profile of solution for fully-coupled eigensolution in FEMLAB. Note the increased temperature gradient near the end and center of the beam.

Fully-coupled simulations were used to predict the Q for flexural beam resonators with slots placed at various locations. The geometry of the device used in the experimental spread is shown in fig. 4. The architecture is a doubly-clamped tuning fork, with two slots in each beam. A tuning fork architecture is used to minimize clamping loss, because the presence of any other energy loss mechanisms will make energy loss due to TED harder to measure. Everything about the resonator architecture, including beam length, beam width, slot length, and slot width, is kept constant, *except* slot location. Parameters used in the simulations are given in Table 1.

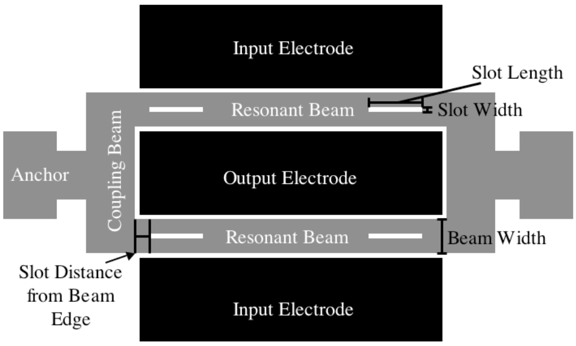

Figure 4. Schematic view of doubly clamped resonator. The resonator consists of two beams coupled together at both ends. Each beam has two slots cut into it a certain distance away from the coupling beam.

Table 1. Material properties and design information for beams with heat interrupting slots.

Parameter	Value	Units
Slot Length	40	μm
Slot Thickness	1	μm
Beam Length	400	μm
Beam Thickness	12	μm
Resonant Frequency	~600	kHz
Density	2330	kg/m^3
Young's Modulus	157	GPa
Poisson's ratio	0.3	
α, CTE	2.6e-6	1/K
Thermal Conductivity	90	W/m/K
Specific Heat	700	J/kg/K
Temperature	300	K

The simulated devices showed a dependence of Q on slot location. Simulations of the beams with different slot locations are shown in fig. 5. The slots have a noticeable impact on the temperature profile. For example, the temperature profile at the center of the beam differs between the beams with slots on the edge and in the center.

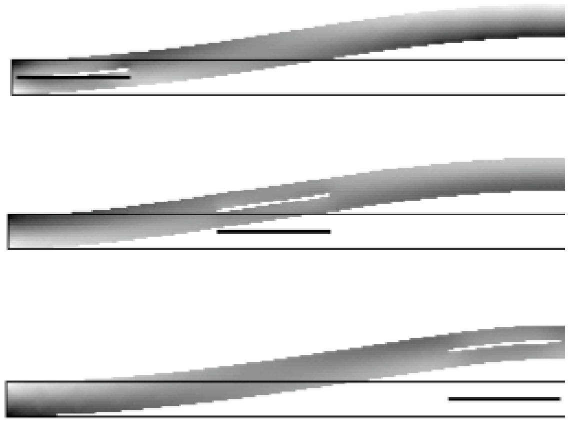

Figure 5. FEMAB fully-coupled solution for 400 μm beam with slots 2 μm, 75 μm, and 158 μm from end of the beam. Temperature profile is shown on deformed beam. Half of each beam is shown.

The devices were fabricated within a wafer-scale vacuum encapsulation [13]. The vacuum (~0.003 mBar) prevents gas damping from limiting the quality factor of the resonator. As an additional check, the resonator with the highest Q was tested in a commercial vacuum chamber to confirm that gas damping was not an energy loss mechanism. Fig. 6 shows a cross section of a resonator beam with slot.

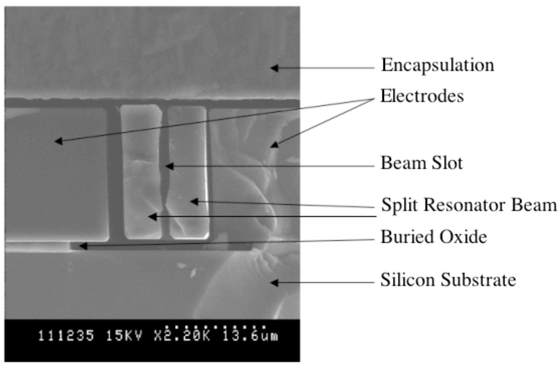

Figure 6. Cross section of encapsulated tuning fork resonator with slot.

The resonators were tested and seen to be in good agreement with the quality factor predicted by simulation, fig. 7. For reference, the Q of a beam of the same dimensions without slots is ~10,300. While the addition of slots improved the Q for all the cases here, this *is not* generally true. The addition of slots can decrease Q, depending on the specific geometry and frequency. This change in Q caused by simply moving the location of the slots stresses the importance of structure design for TED-limited quality factor. It is important to point out that, while some intuition can be useful in placing slots near areas of high strain, it does not provide all the information necessary. The coupling between the thermal and mechanical modes is very complex, and the fully-coupled simulations are necessary for an accurate prediction of TED-limited Q. It is not accurate to estimate multiple thermal time constants across the beam and apply Zener's equations to each time constant. The work given here shows that there is more complexity than two critical dimensions across the beam, because the placement of the slots along the length of the beam affects Q.

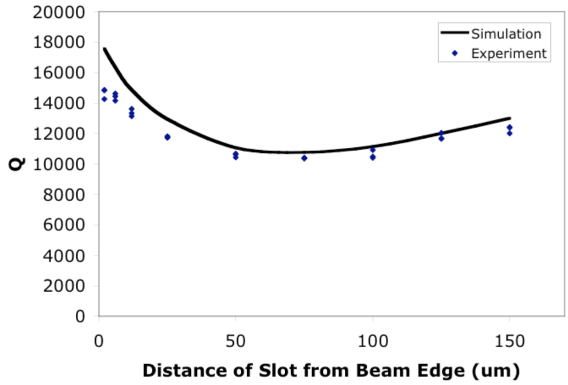

Figure 7. Comparison of simulation to experimental results for resonators with the same geometry, *except* for the location of the heat interrupting slots. Experimental data confirms that the location of slots on resonant beams has an impact on quality factor.

CONCLUSIONS

Effects of resonator geometry on TED-limited Q were explored. Specifically, slots were micromachined into flexural beams to alter the coupling between the mechanical and thermal domains. The complexity of this coupling, as evidenced by the dependence of Q on slot placement, makes the use of amendments to Zener's Q equation insufficient for arbitrarily complex geometries. The TED-limited Q was shown to be a function of the slot location along the beam, with slots at regions of high strain having the greatest impact.

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