TEMPERATURE DEPENDENCE OF QUALITY FACTOR IN MEMS RESONATORS

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ABSTRACT

The temperature dependence of the quality factor, Q, of encapsulated MEMS resonators is analyzed in an effort to understand the temperature regimes where different energy loss mechanisms are dominant. The effect of two limiting energy loss mechanisms for these resonators, air damping and thermo elastic dissipation, are separately analyzed to determine the Q of the system over a range of temperatures. MEMS resonators can be designed to have either strong weak dependence of Q on temperature, if the effects of the dominant loss mechanisms with temperature are well understood. Up to 1% change in quality factor per °C change of temperature was demonstrated, leading to the possibility of using quality factor as an absolute thermometer for temperature compensation in MEMS resonators.

1. INTRODUCTION

MEMS resonators are a promising technology for the replacement of quartz resonators in frequency reference applications. Reduced size and CMOS integration of silicon MEMS resonators are expected to result in increased capability, miniaturization of electronic devices and reduction in power consumption. Moreover, recently developed wafer-scale encapsulation technologies may lead to significant cost reduction and enhanced stability by avoiding higher level packaging [1].

Quality factor is one of the most important features in MEMS resonators, because a significant high Q (>10,000) is required to guarantee stable performance of local oscillators or synchronizing clocks in transceivers [2]. However, the temperature dependence of the quality factor has to be carefully investigated because of the following reasons. First, some efforts to overcome the temperature coefficient of the resonant frequency (TCf) are performed by local heating of MEMS resonator above environmental temperature. This means that high quality factor should be achieved at much higher temperature (*e.g.* 125°C). Second, strong temperature dependency of the quality factor may allow direct measurement of temperature so that it may be used as a method for temperature compensation.

f	resonant frequency	Q	Quality factor
ρ	density	m	gas molecule mass
T	temperature	p	pressure
k_b	Boltzmann constant	h	resonator beam width
α	thermal expansion coeff.	C_s	specific heat
E	Young's modulus	k	thermal conductivity

Table 1. Symbols used in this study

2. QUALITY FACTOR IN MEMS RESONATORS

Quality factor is defined as the energy stored in the system divided by the energy dissipated per radian. There has been analysis and demonstration of various mechanisms that may yield energy loss of MEMS resonators. Each mechanism can be considered as an individual Q limiting value, with the lowest individual Q (largest energy dissipation) dominating the overall Q of the resonator,

$$\frac{1}{Q_{total}} = \frac{1}{Q_{air}} + \frac{1}{Q_{TED}} + \frac{1}{Q_{anchor\,loss}} + \frac{1}{Q_{others}} \quad (1)$$

AIR DAMPING

Air damping is one well known energy loss mechanism for micro scale structures. When micro structures are moving, collisions with air particles yield energy loss [3]. The quality factor of micro scale resonators due to air damping in the kinetic gas regime is given as,

$$Q_{air} = \frac{h(\pi/2)^{5/2}\rho f\sqrt{\frac{k_b T}{m}}}{p} \propto \frac{\sqrt{T}}{p} \quad (2)$$

Figure 1 shows quality factor of MEMS resonators vs. pressure. At higher pressure, quality factor is inversely proportional to the pressure, which indicates air damping is the dominant energy loss mechanism. However, at lower pressure, the quality factor is not a function of pressure, because other energy loss mechanisms become dominant. By performing a pressure sweep, Q_{air} can be factorized.

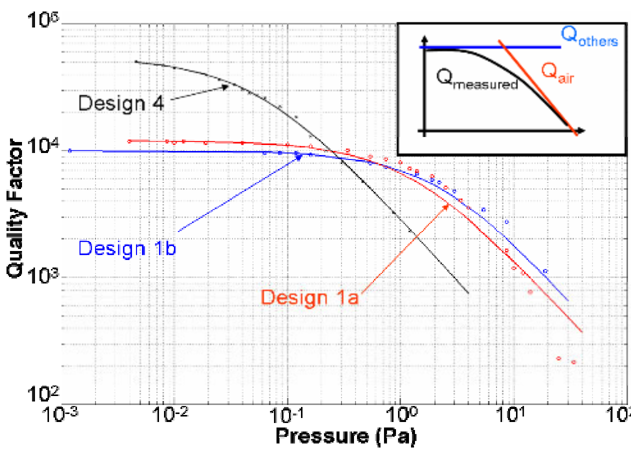

Figure 1 Plot of quality factor with respect to pressure. Q_{air} can be estimated from the Q in the pressure limited regime(at higher pressure) of this plot as shown in the schematic of top right corner.

THERMOELASTIC DISSIPATION

Another key energy loss mechanism is thermoelastic dissipation (TED). When the resonator beams flex, strain gradients will be produced, and this leads to temperature gradients. Difference in temperature yields thermal transport from hotter to colder area, and energy is dissipated through this mechanism. If the time constant of thermal transport is close to the deflection period of the beam the energy loss is maximized, resulting in low Q.

$$Q_{TED} = \left(\frac{f_M^2 + f_T^2}{f_M f_T} \right) * \frac{C_p \rho}{\alpha^2 TE} \quad (3)$$

$$where,\ f_T = \frac{\pi k}{2 C_p \rho h^2}$$

This phenomenon is well described by the above expression for simple, rectangular beams, and can be modeled for other geometries [4, 5].

OTHER ENERGY LOSS MECHANISMS

There are several more energy loss mechanisms known in microscale resonators. Anchor loss is expected to contribute at higher frequencies, but a quantitative predictive model is not available [6]. Among resonators investigated in this study, two kinds of resonators with the same geometry but with only different anchoring were included (*i.e.* one was anchored on one side and the other was anchored on the both sides of the resonator beam). Because there was no difference in quality factor between the two types of anchors, anchor loss is not thought to be a dominant energy loss mechanism in these tuning fork resonators.

Surface loss is another known energy loss mechanism in microscale resonators. However, since our Q values have been accurately predicted by TED and air-damping models, we do not believe that other loss mechanisms are contributing significantly to Q in our resonators.

3. TEMPERATURE DEPENDENCE OF QUALITYF FACTOR

As discussed earlier, total quality factor value itself is determined by the smallest value among each factorized quality factor. Also, temperature dependence of total quality factor is primarily determined by that of the dominant energy loss mechanism (smallest Q) in the case where multiple energy loss mechanisms exist. This phenomenon is explained in figure 2. For better understanding, we introduce a coefficient that indicates temperature dependency of quality factor, called *Temperature Coefficient of Quality Factor* (TCQ). This TCQ is defined as the exponential in equation 4.

$$Q \propto \frac{1}{T^{\gamma}}, \quad \gamma = TCQ \quad (4)$$

But if two mechanisms are equally or significantly dominant, total TCQ will have a value between the TCQs of each. Therefore, to understand temperature dependence of quality factor, we have to investigate the temperature dependence and weight of each loss mechanism.

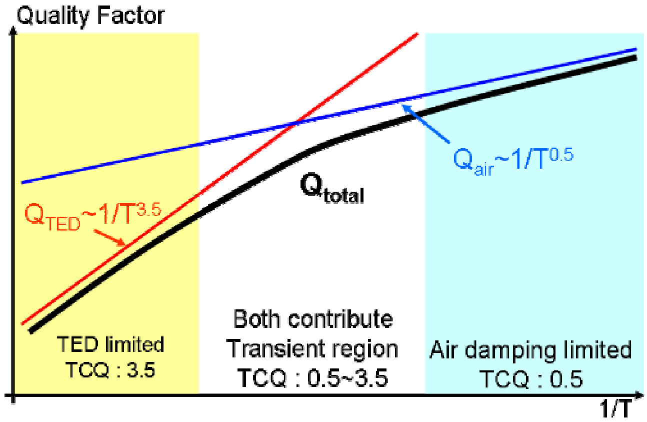

Figure 2 Concept schematic for total quality factor. The sample TCQ for TED is used 3.5. If TED is the dominant energy loss TCQ will approach to 3.5, whereas air damping dominant, TCQ will become closer to 0.5. In the transient region, TCQ varies between 0.5 to 3.5

TEMPERATURE DEPENDENCE OF Q_{air}

As equation 1 suggests, at given pressure, Q_{air} (quality factor for air damping) is proportional to square root of temperature. However, once the resonators are encapsulated, the pressure is no longer a constant value with varying temperature. Instead, if diffusion through encapsulation is negligible, the number of molecule in the cavity is fixed, and pressure itself becomes proportional to the temperature. Assuming ideal gas, which is good assumption in epi-sealed case because the only gas in the cavity is hydrogen, Q_{air} becomes,

$$Q_{air} \propto \frac{\sqrt{T}}{p} \propto \frac{\sqrt{T}}{n k_b T} \propto \frac{1}{\sqrt{T}} \quad (5)$$

Therefore, TcQ_{air} is 0.5 for encapsulated resonators.

TEMPERATURE DEPENDENCE OF Q_{TED}

At first glance, Q_{TED} from equation 3 may suggest TCQ_{TED} = 1. However, material parameters change very rapidly even in normal operation temperature range (250~400K), so the resulting temperature dependence is very complicated. Figure 3 shows the change in material parameters in the normal operation temperature range calculated by Debye theory. With consideration of this change, temperature dependence of Q_{TED} should be carefully investigated. For better understanding, we can split equation 3 two parts, one is called frequency term, $Q_{TED,freq}$ and the other is material term, $Q_{TED,mat}$, and total Q_{TED} is just the product of these two.

$$Q_{TED} = \left(\frac{f_M^2 + f_T^2}{f_M f_T} \right) * \frac{C_p \rho}{\alpha^2 TE} = Q_{TED,freq} Q_{TED,mat}$$

$$where,\ Q_{TED,freq} = \frac{f_M^2 + f_T^2}{f_M f_T}, \quad f_T = \frac{\pi k}{2 C_p \rho h^2} \quad (6)$$

$$Q_{TED,mat} = \frac{C_p \rho}{\alpha^2 TE}$$

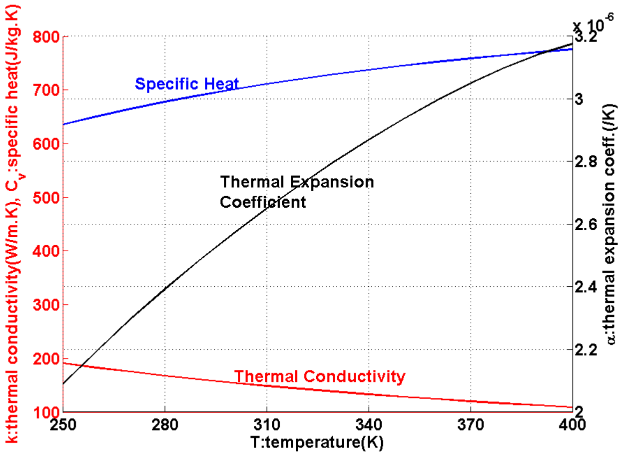

Figure 3 Material properties of silicon at normal operating temperature (250K-400K). Because the doping concentration of used silicon is low, pure silicon properties are used for calculation of quality factors.

At a given mechanical resonant frequency, $Q_{TED,freq}$ decreases as thermal mode frequency, f_T approaches closer to mechanical mode frequency, f_M. The beam height, h, is design dependent, but all other parameters to decide f_T are material values and don't depend on design. Interestingly, the material parameters always cause f_T to decrease as temperature increases at normal operating temperature (Fig.4). Therefore, whether $Q_{TED,freq}$ increases or decreases with increasing temperature depends on whether f_T is larger or smaller than f_M at room temperature. If f_T is larger than f_M, $Q_{TED,freq}$ will decrease with increasing temperature, because f_T approaches f_M. When $f_T = f_M$, $Q_{TED,freq}$ is a minimum. Conversely, $Q_{TED,freq}$ increases with increasing temperature when f_M is larger than f_T.

At the same time, $Q_{TED,,mat}$ decreases rapidly as temperature increases. Figure 5 illustrates the effect of increased temperature for Q_{TED} which is the product of $Q_{TED,freq}$ and $Q_{TED,,mat}$. As shown here, even two different resonator designs with the same mechanical resonant frequency and the same Q_{TED} at some temperature but different beam heights may yield different temperature dependency of Q_{TED}

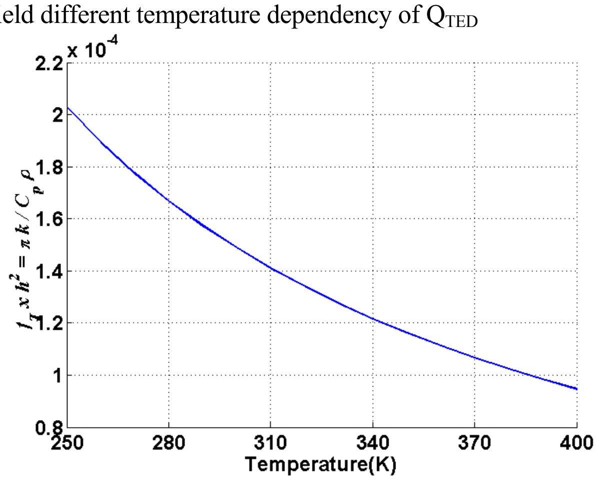

Figure 4 Plot of pure material values in thermal mode resonant frequency vs temperature. As shown in this plot, regardless dimensions of resonators, thermal mode resonant frequency always decreases as temperature increases.

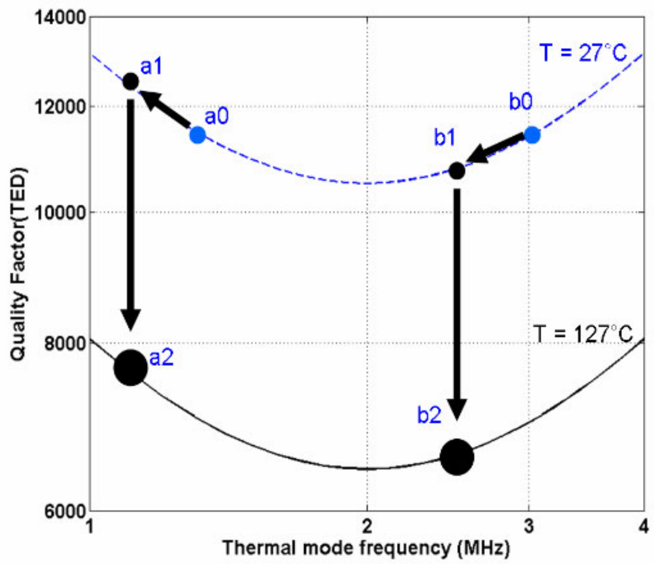

Figure 5 Example plot of how quality factor changes as temperature increases. Both a0 and b0 represents two different resonator designs which have Q~11,000 and f_M=2MHz at room temperature (27ºC). If temperature increases because thermal mode frequency, f_T, shifts left, quality factor will become a1 and b1 respectively. However increase in temperature will yield shift in the curve itself. Finally, status of two designs will become a2 and b2 respectively, showing different quality factor at high temperature.

4. EXPERIMENTS

Several tuning fork type MEMS resonators with various dimensions were used for the experiment (fig.6). All the resonators were encapsulated by our epi-seal process which leaves only less than 1 Pa hydrogen in the cavity and privides a perfect hermetic seal [7, 8]. To experimentally evaluate the temperature dependence of quality factor, over 20 temperature cycles from -20 to 80°C of 15 resonators were performed within a Thermotron S1.2 temperature chamber. Quality factors and resonant frequencies of each resonator were measured by Agilent4395A network analyzer every 10°C.

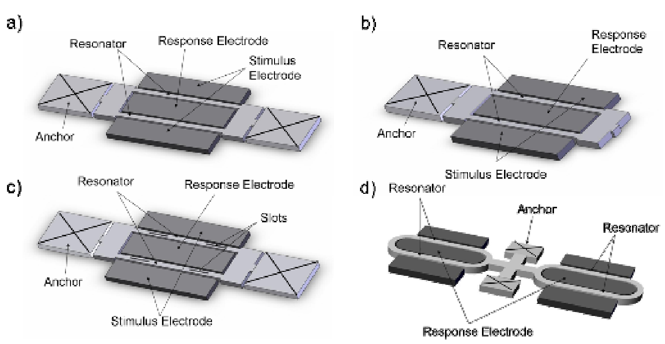

Figure 6 Schematic of resonator designs used for the test. 1. a) double anchor, b) single anchor, c) double anchor – slotted to reduce thermo elastic dissipation, d) single anchor

Design	f_M + (kHz)	f_T + (kHz)	Shape (Fig.6)	# in the test	Q_{air} + (Estimated)	Q_{TED} + (Calculated)	TCQ_{TED} (Theory)	Q_{other} + (Estimated)	Q + (Measured)	TCQ_{Total} (Measured)
1a	~1,300	~2,300	a	3	~50,000	~12,300	3.1	~100,000	~9,000	2.9 ~ 3.8
1b	~1,300	~2,300	b	4	~50,000	~12,300	3.1	~100,000	~9,000	2.9 ~ 3.4
1c	~1,300	~2,300	a	2	~50,000	~12,300	3.1	~100,000	~9,000	3.4~3.7
2	~590	~1,030	c	2	~34,000	~12,200	3.1	~18,000	~6,000	2.1 ~ 2.2
3	~2,100	~1,030	a	1	~115,000	~13,200	1.3	TED limited	~13,000	1.7
4	~155	~4,130	d	3	~68,000	~140,000	4.0	~400,000	~30,000	1.3 ~ 2.3

Table 2. Characteristics of resonators used in the test. Based on the fact that the smaller Q determines total Quality factor, Design 1 and Design 3 are TED limited which show TCQ close to TCQ_{TED}, while Design 4 which Q_{TED} dominant has TCQ close to TCQ_{air}. Design 2 in which both are similarly dominant, has TCQ in the middle. + Measured in room temperature. 1c is attached to the package by hard epoxy..

5. RESULTS

As seen in figure 7, both single and double anchored resonators showed very similar TCQ. Because both double and single anchored resonators showed almost same TCQ, it shows either anchor loss is not temperature dependent or the effect is not dominant compared to other loss mechanisms. different temperature dependency of quality factor. Design 1 had larger f_T (~2.3MHz) than f_M (~1.3MHz) at room temperature. So, with increased temperature f_T (~1.5MHz at 125°C) approached toward f_M, and quality factor dropped rapidly. At the same time, in the case of Design 3, because room temperature f_T (~1.0MHz) was smaller than f_M, f_T became father (~0.7MHz at 125°C) and quality factor dropped slower.

6. CONCLUSIONS

One important result is that it is possible to design resonators that have highly temperature dependent quality factor by selecting the position of the thermal and mechanical relaxation times of the resonators. Since the limitation of quality factor by TED is highly dependent on the beam geometry and temperature dependence of material parameters,

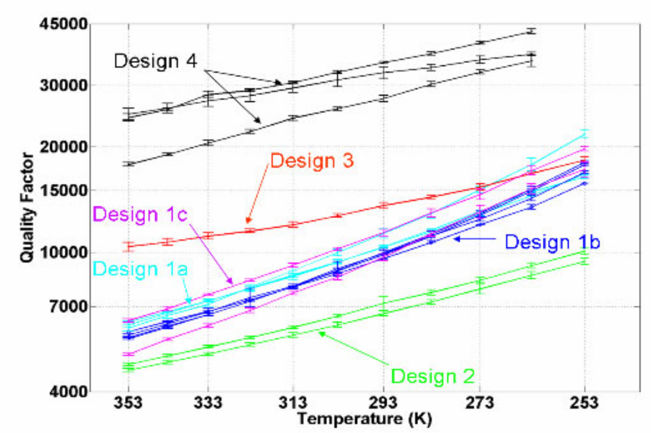

Figure 7 Log plot of quality factor with respect to temperature for all the 15 resonators used. The slope of Q vs 1/T line (TCQ) indicates the dominant energy loss mechanism for the specific resonators. The closer TCQ value to the theoretical value of the energy loss mechanism, the more dominant that mechanism to decide total quality factor as shown in Table 2.

the possibility of optimized design exists, wherein the quality factor, TCQ, and resonant frequency can all be simultaneously optimized. TCQ values as high as 3.5 have been demonstrated. These resonators show quality factor change of almost 1 % per degree of temperature change. These resonators have been shown to have drift in quality factor of much less than 5% over a year in long-term test [7]. Therefore, Q(T) is potentially a very useful absolute thermometer for temperature compensation in MEMS resonators.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

[1] W.-T. Park, et al, "Wafer Scale Encapsulation of MEMS Devices," INTERPACK'03 The Pacific RIM/ASME, Maui, Hawaii, USA, 2003.

[2] C. Nguyen, "High-Q micromechanical oscillators and filters for communications," IEEE ISCAS' 97. Circuits and Systems in the Information Age, Hong Kong, 1997.

[3] W. E. Newell, "Miniaturization of Tuning Forks," *Science*, pp. 1320-1326, 1968.

[4] R. N. Candler, et al, "Impact of Geometry on Thermoelastic Dissipation in Micromechanical Resonant Beams," *Journal of MEMS*, in review, 2005.

[5] R. N. Candler et al, "Investigation of energy loss mechanisms in micromechanical resonators," presented at IEEE Int'l Solid-State Sensors and Actuators Conference, Boston, USA, 2003.

[6] Z. Hao and F. Ayazi, "Support Loss In Micromechanical Disk Resonators," presented at MEMS 2005. 18th IEEE International Conference on MEMS, Miami, FL, USA, 2005.

[7] B. Kim, et al, "Investigation of MEMS resonator characteristics during long-term and wide temperature operation," presented at 2004 ASME Int'l Mechanical Eng. Congress and RD&D Expo, Anaheim, CA, USA, 2004.

[8] R. N. Candler et al, "Long-Term and Accelerated Life Testing of a Novel Single-Wafer Vacuum Encapsulation of MEMS Resonators," *Journal of MEMS,* in review, 2005.