What is the Young's Modulus of Silicon?

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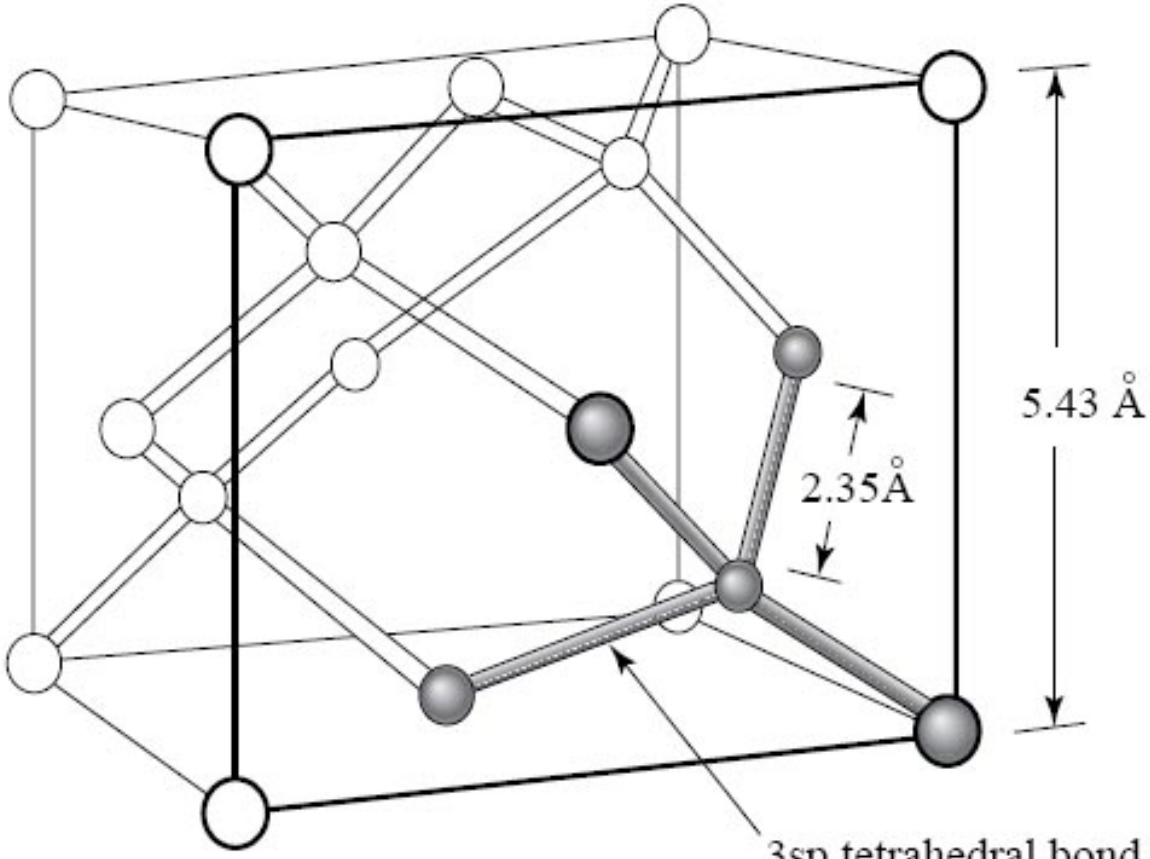
Silicon has a regular crystal structure, which is one of the reasons it is such an excellent engineering material. It is an anisotropic crystal, so its properties are different in different directions in the material relative to the crystal orientation.

When designing mechanical structures in silicon, it is important to understand which crystal orientations are relevant to the structure and what corresponding values for mechanical properties should be used for calculations.

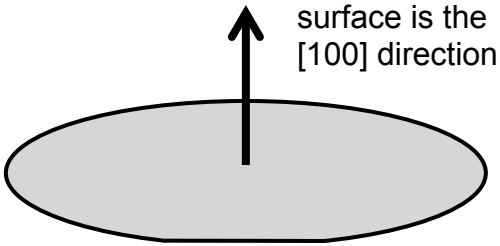


http://onlineheavytheory.net/silicon.html

These slides are only a summary. For complete information, see:
M. A. Hopcroft, W. D. Nix, and T. W. Kenny, "What is the Young's Modulus of Silicon?," Journal of Microelectromechanical Systems, vol. 19, pp. 229-238, 2010.

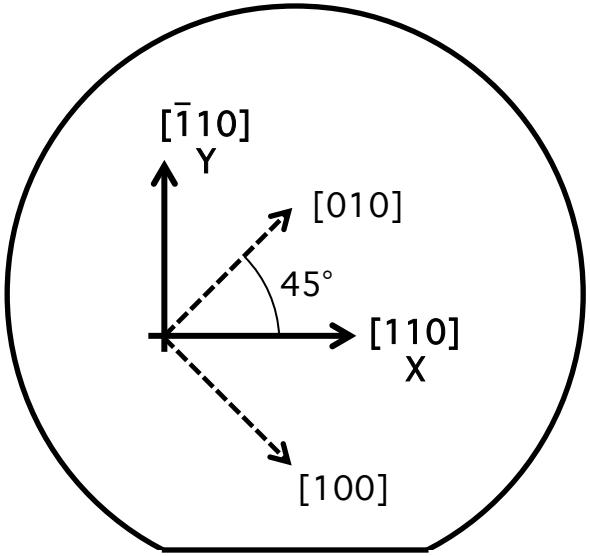
What is the Crystal Orientation in a Silicon Wafer?

The crystal orientation of a wafer is defined by the plane of its top surface. (100) wafers are most common, but other orientations are available.



The direction normal to the top surface is the [100] direction

The top surface of a "(100) wafer" is the (100) crystal plane



A typical (100) wafer with in-plane directions indicated

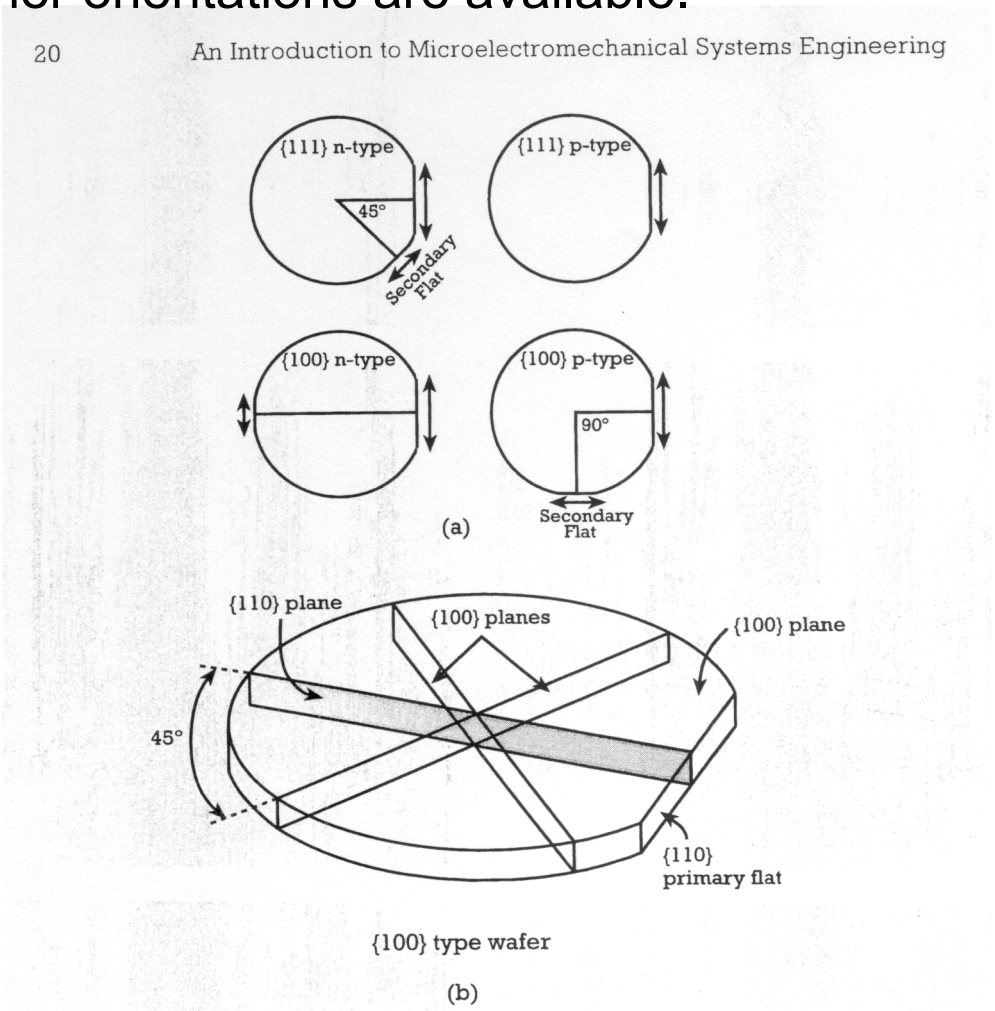


Figure 2.2 (a) Illustration showing the primary and secondary flats of {100} and {111} wafers for both *n*-type and *p*-type doping (SEMI standard). (b) Illustration identifying various planes in a wafer of {100} orientation.

N. Maluf, An Introduction to Microelectromechanical Systems Engineering, 1st ed. Boston: Artech House, 2000.

Fine. So What Value Should I Use for *E* of Silicon?

• Silicon is an anisotropic crystal, so its Young's modulus varies in different directions in the material relative to the crystal orientation.

• Silicon has cubic symmetry, so the 3D direction-dependent properties can be described with a 6x6 matrix with only 3 independent constants (either stiffnesses C_{ij} or compliances S_{ij}). Because silicon is such an important economic material, these values have been investigated thoroughly. The best values of these constants are:

C: 10^9 Pa S: 10^{-12} Pa	C_{11}	C_{12}	C_{44}	S_{11}	S_{12}	S_{44}
Si	165.7	63.9	79.6	7.68	-2.14	12.6

W. A. Brantley, "Calculated elastic constants for stress problems associated with semiconductor devices," Journal of Applied Physics, vol. 44, pp. 534-535, 1973.

J. J. Wortman and R. A. Evans, "Youngs' Modulus, Shear Modulus and Poisson's Ratio in Silicon and Germanium," Journal of Applied Physics, vol. 36, pp. 153-156, 1965.

H. J. McSkimin and J. P. Andreatch, "Elastic Moduli of Silicon vs Hydrostatic Pressure at 25.0 。C and - 195.8 。C," Journal of Applied Physics, vol. 35, pp. 2161-2165, 1964.

W. P. Mason, Physical acoustics and the properties of solids. Princeton, NJ USA: Van Nostrand, 1958.

• For FEM calculations, use the stiffness matrix. Be sure to type it in yourself! Many common programs (ANSYS, FEMLAB) do not have default values.

For design calculations:

• Axial tension/compression: Use the *E* value for the direction of tension/ compression.

$$\frac{1}{E_{\alpha\beta\gamma}} = S_{11} - 2\left[\left(S_{11} - S_{12}\right) - \frac{1}{2}S_{44}\right]\left(l^2m^2 + m^2n^2 + l^2n^2\right)$$

$$l = \cos(\alpha)^*$$
$$m = \cos(\beta)$$
$$n = \cos(\gamma)$$

$$\frac{1}{E_{100}} = S_{11} \qquad \frac{1}{E_{110}} = S_{11} - \frac{1}{2}\left[\left(S_{11} - S_{12}\right) - \frac{1}{2}S_{44}\right] \qquad \frac{1}{E_{111}} = S_{11} - \frac{2}{3}\left[\left(S_{11} - S_{12}\right) - \frac{1}{2}S_{44}\right]$$

J. F. Nye, Physical properties of crystals : their representation by tensors and matrices. Oxford: Oxford University Press, 1985.

* "direction cosines": cosine of the angle between the direction of interest and x,y,z axes (the <100> directions)

• For example, in a (100) wafer:
  • for “x or y axis” (parallel to flat), use E_{110} = 169 GPa
  • for “off-axis” (45° diagonal to flat), use E_{100} = 130 GPa

•Small deflection of a long, thin beam: This is basically axial tension and compression, so use the *E* value for the direction of the neutral axis.

• Small deflection of a plate edge: Use a “plate modulus” for the major axis of bending: $\frac{E_{\alpha\beta\gamma}}{1-\nu}$, with Poisson’s ratio of ν = 0.064 for <110> in (100). [Brantley 1973]

• Thin film stress/strain calculations for a thin film membrane or bending substrate (incl. Stoney's equation calculations): Use the symmetric Biaxial modulus B_{ijk} for the appropriate symmetric crystal plane, either (100) or (111) (the (110) plane is not symmetric).

$$B_{100} = C_{11} + C_{12} - \frac{2C_{12}^{\ 2}}{C_{11}} \qquad B_{111} = \frac{6C_{44}\left(C_{11} + 2C_{12}\right)}{C_{11} + 2C_{12} + 4C_{44}}$$

For example, B_{100} = 179.4 GPa W. D. Nix, Stanford MSE 353 Material Properties of Thin Films

• For stress concentration calculations or multi-directional polycrystalline situations, use a Voigt (172 GPa) or Reuss (164 GPa) volume average.

J. Diz and M. Humbert, "Practical aspects of calculating the elastic properties of polycrystals from the texture according to different models," Journal of Applied Crystallography, vol. 25, pp. 756-760, 1992.
W. N. Sharpe, Jr., B. Yuan, and R. L. Edwards, "A new technique for measuring the mechanical properties of thin films," Journal of Microelectromechanical Systems, vol. 6, pp. 193-199, 1997.

• For hydrostatic loads, use a Bulk modulus, B = 97.83 GPa.

George, "Elastic constants and moduli of diamond cubic Si," in Properties of Crystalline Silicon, vol. 20, EMIS Datareviews, R. Hull, Ed. London: INSPEC, IEE, 1997, pp. 98.

• Most silicon wafers are not pure silicon. Electronic doping changes the elastic behaviour in predictable ways. Typically the changes are negligible (1-5%).

See J. J. Hall, "Electronic effects in the constants of n-type silicon," Physical Review, vol. 161, pp. 756, 1967, and references cited therein.

For more information, see:

•H. J. McSkimin, "Measurement of Elastic Constants at Low Temperatures by Means of Ultrasonic Waves--Data for Silicon and Germanium Single Crystals, and for Fused Silica," Journal of Applied Physics, vol. 24, pp. 988-997, 1953.
•H. J. McSkimin, W. L. Bond, E. Buehler, and G. K. Teal, "Measurement of the Elastic Constants of Silicon Single Crystals and Their Thermal Coefficients," Physical Review, vol. 83, pp. 1080, 1951.
•C. Bourgeois, E. Steinsland, N. Blanc, and N. F. de Rooij, "Design of resonators for the determination of the temperature coefficients of elastic constants of monocrystalline silicon," Proceedings of the 1997 IEEE International Frequency Control Symposium, 1997.
•Stanford MSE 353 (Nix / Barnett)
•Stanford MSE 208 (Dauskart)